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The Design, Experimental Laboratory Testing and Implementation of a Large, Multi-Market, Policy Constrained, State Gaming Machines Auction

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Abstract: The paper reports on the theory, design, laboratory experimental testing, field implementation and results of a large, multiple market and policy constrained auction. The auction involved the sale of 18,788 ten-year entitlements for the use of electronic gaming machines in 176 interconnected markets to 363 potential buyers representing licensed gaming establishments. The auction was conducted in one day and produced over $600M in revenue. The experiments and revealed dynamics of the multi-round auction provide evidence about basic principles of multiple market convergence found in classical theories of general equilibrium using new statistical tests of the abstract properties of tatonnement.

Section 1: Introduction[1]

The paper reports on the design, laboratory experimental testing, and field implementation of a large, multiple market and policy constrained auction. The auction involved the sale of 18,788 ten-year entitlements for the use of electronic gaming machines in Victoria Australia, in May, 2010. Policy issues dictated the operation of 176 interconnected markets to allocate sales of these licenses to 363 potential buyers representing licensed gaming establishments. The auction was conducted in one day and produced over $600M in revenue. The design rested on basic principles of competitive economics for a general equilibrium exchange economy guided by the classical tatonnement model of market adjustment. The theoretical framework in which we interpret the results is informed by competitive market principles that demonstrate convergence to an equilibrium with many features predicted by the classical theory. The paper reviews the policy background, the theoretical architecture, a discussion of key features of the laboratory experimental testing and discussions of results and dynamic performance.

Two overriding research questions are addressed by the paper. The questions are posed as broad guidelines for assessing the success of policy related institutional and market designs (Plott, 1994). First, was the implementation successful in satisfying the goals the policy was

[1] The financial support for analyzing the data and developing this report provided to Plott by the Rising Tide Foundation (Grant Number: RTF-19-500) and the John Templeton Foundation (Grant Number: 58067) are gratefully acknowledged as is the technical support of the Caltech Laboratory for Experimental Economic and Political Science. Limited data analysis for scientific purposes was allowed by the Victorian government. The cooperation and help of William Stevenson of Intelligent Market Systems LLC were fundamental.

designed to meet? Secondly, was the science applied to guide the design successful in attributing the outcomes to the principles used in the design? As it turns out the auction outcomes satisfied all policy constraints. Furthermore, the auction provides substantial support for the broad reliability of the basic principles of classical, competitive economic theory that support the auction architecture and guided the design.

The auction was the result of the Victorian government's efforts to implement a major change in the policies regarding gambling operations in the state of Victoria. In 2008, the government initiated a reorganization of this industry by changing the method of allocating the entitlements to operate electronic gaming machines (e.g. poker and slot machines) and the method of finance. Historically, the distribution of gaming machines was managed by two large corporations associated with the machine manufacturers. The machines were allocated to businesses consistent with local policies governing their use. Finance was based on the revenue produced by the machines with roughly a third going to the local establishment, a third to the managing company, and a third to the government.

Governmental concerns with the historical policy reflected a desire for better control over gambling and concomitant social problems, gambling related government public finance, and a desire for conformity with frameworks used for economic regulation. The transition policy chosen was based on an auction intended to accommodate several broad economic objectives. The policy objectives did not include revenue maximization, which would have justified a monopolistic supply determination that was deemed outside the auction design problem. The demand for entitlements was to be discovered by the auction process and the aggregate supply was dictated to be near historical levels by policy. The auction was intended to allow fluid price discovery and thus allocate entitlements smoothly and efficiently in a manner consistent with a competitive market. The goal was to create minimal economic dislocations and a climate where future regulatory efforts could be based on competitive principles of decentralized competition and profits. The auction was also implemented to allow for possible entry and shifting of entitlements from past use to reflect underlying economic value rather than being based on historical administrative practices.

The resulting mechanism and its implementation present a remarkable success for the decades of abstract theorizing about general equilibrium in classical economics. As demonstrated in Section 2's presentation of the auction structure, this theory proved to be quite useful in practice when defining the mechanism. Further, Section 3 demonstrates how lessons learned from testbedding experiments in a laboratory environment were able to scale to field implementations. Partial equilibrium analysis in Section 4 illustrates the operation of market principles supporting an efficient allocation of licenses within each market.

Evaluating the overall allocation of licenses across markets and verifying the auction reached an efficient general equilibrium presents a challenging empirical exercise. To this end, we introduce an important principle, "excess demand revealed at the margin," that can be readily measured from observed bidding behavior. Section 5 demonstrates that this excess demand is exhausted through the auction mechanism's bid revision process. We further explore the dynamic properties of the auction mechanism in Section 6, characterizing the total revenues and surplus generated by the auction mechanism. Section 7 investigates the relationship of price dynamics across markets to the revealed excess demand in all other markets. This analysis verifies the conditions for stability that would lead to an efficient multi-market allocation and general equilibrium across all market segments given policy constraints. These novel statistical tests take advantage of the rich data we have available and present the first empirical verification of equilibrating dynamics based on the principles of tatonnemont. Section 8 concludes, presenting a summary of the findings from the implementation of an economic mechanism to address the allocation problem at the heart of a complex government policy project.

Section 2. Auction Structure

Section 2.1 Policy Constraints

The auction design problem was to simultaneously sell rights subject to many overlapping policy constraints. Key policy constraints were focused on the nature of the businesses that were allowed to participate in the auction and acquire entitlements. Half of the 27,500 entitlements were to be sold to businesses classified as Hotels, which were larger venues possessing substantially greater value for the gaming machines. The other half was to be sold to smaller venues called Clubs that cater to local populations. This reflected differences between the economic environment and social purposes of these venues and differing political bases in the Victorian communities.

For purposes of the allocation Victoria was divided into 88 geographic regions, and each region had a maximum number of entitlements based on area population or other regulations.[2] These constraints placed limits on the saturation of machines relative to population and were motivated, in part, by social and health issues related to gambling. Additional policy concerns related to the geographic distribution of entitlements resulted in the creation of a single set geographic regions designated as metropolitan and maximum number of entitlements that could be allocated to the set.

Accounting for these constraints, and neglecting the metropolitan designation, each entitlement has two characteristics: the type of venue (Club or Hotel) and the geographical

[2] Additionally, geographical areas designated as "metropolitan" were limited to obtain no more than 80 percent of all entitlements. Since this constraint did not bind at any point in the system, we avoid discussing the features implemented to accommodate the constraint should it be binding.

region (88 distinct areas) in which that establishment is allowed to operate. This required 176 simultaneous markets.

2.2 Determining Allocations and Prices in a Continuous Model

The basic auction design can best be understood in the context of a continuous model that assumes away complexities created by underlying integers. These complexities will be addressed in the later sections that analyze the data from the auction.

As introductory notation, let $i \in I$ index each establishment and let $a_i \in A$ denote the area in which the establishment seeks to obtain licenses.[3] The indicator variable $h_i \in \{0,1\}$ identifies the i^{th} establishment type, equaling 1 if establishment i is a hotel.

Suppose that each bidder submits a continuous valuation schedule $V_i(x)$ reporting their total willingness to pay for an allocation of x entitlements. As an additional regularity condition, suppose further that $D_i(x) = \frac{\partial V_i(x)}{\partial x}$, representing the bidder's marginal willingness to pay is non-negative, monotonic, and (weakly) decreasing.[4] These conditions ensure that the representation of bidders' demand schedules for licenses is continuous and monotonically weakly decreasing.

The system allocates entitlements to bidders so as to maximize the total cumulative reported value of the allocation. Define X as a vector of allocations with the i^{th} entry x_i representing the allocation to establishment i. We measure the total market value, $V(X)$, as the aggregate value of bidders' willingness to pay for their given allocations:

$$V(X) = \sum_{i \in I} V_i(x_i)$$

[3] Each establishment corresponds to a policy-defined "venue" meaning the location where the machines would be housed and operated. Bids are submitted by the venue and the entitlement is issued to the venue where the machine must be located and counts against the area constraints. A business might own more than one venue and employ a representative bidder authorized to submit bids for more than one venue that the business might own. Auction rules were designed to minimize coordination between bidders. All bidders were located in a large room with cubicles from which other bidders could not be viewed. To constrain the information that bidders receive, external communication devices were prohibited and monitors were tasked with ensuring no unauthorized communication occurred amongst the bidders. While a representative bidder might tender bids for all of the venues from the same terminal each bid is attached to a specific venue and recorded as made by that venue for purposes of policy consistency. As such, even though a single business might own more than one venue even in one particular area, the system records and treats each venue as a separate entity. Special rules and monitoring were imposed for multiple venues operating under the same ownership.

[4] We will discuss the mechanics for eliciting bidders' demand and valuations in the next subsection. In discussing implementation, we will also review some complexities that were overcome when mapping the continuous model onto a discrete price and commodity space.

As discussed in the previous section, these allocations must satisfy the set of policy constraints defined by the government of Victoria. Administering these constraints is facilitated by defining the total allocations to each area by venue type:

$$x_{aC} = \sum_{i \in I} x_i \left(1 - h_i\right) 1\left\{a_i = a\right\} \qquad x_{aH} = \sum_{i \in I} x_i h_i 1\left\{a_i = a\right\} \qquad x_a = x_{aC} + x_{aH}$$

Imposing these definitions as equality constraints on the maximization problem identifies the shadow costs for allocating a marginal license to each area and bidder type. The total allocation to an area, x_a, must satisfy the constraint defined by the Victoria government, denoted $\overline{x}_a$: $x_a \le \overline{x}_a,\ \forall a \in A$.

Administering Victoria-wide constraints on the total allocations to Hotels and Clubs, respectively denoted $\overline{x}_H$ and $\overline{x}_C$, is facilitated by similar constraints:

$$x_H = \sum_{a \in A} x_{aH} \qquad x_C = \sum_{a \in A} x_{aC}$$

Each of these aggregated allocations must satisfy government-imposed inequality constraints so that $x_H \le \overline{x}_H$ and $x_C \le \overline{x}_C$.

The Lagrangian for the constrained allocation problem is:

$$\begin{aligned}
\mathscr{L}\left(X\right) = V\left(X\right) & \\
& \left.\begin{aligned}
&-\sum_{a \in A} \lambda_{aC}\left(x_{aC} - \sum_{i \in I} x_i \left(1 - h_i\right) 1\left\{a_i = a\right\}\right) \\
&-\sum_{a \in A} \lambda_{aH}\left(x_{aH} - \sum_{i \in I} x_i h_i 1\left\{a_i = a\right\}\right) \\
&-\sum_{a \in A} \lambda_a\left(x_a - x_{aC} - x_{aH}\right) \\
&-\sum_{a \in A} \mu_a\left(\overline{x}_a - x_a\right)
\end{aligned}\right\} \text{Constraints on area Club and Hotel allocations} \\
& \left.\begin{aligned}
&-\lambda_H\left(x_H - \sum_{a \in A} x_{aH}\right) - \lambda_C\left(x_C - \sum_{a \in A} x_{aC}\right) \\
&-\mu_H\left(\overline{x}_H - x_H\right) - \mu_C\left(\overline{x}_C - x_C\right) \\
&-\mu_O\left(\overline{x} - x_H - x_C\right)
\end{aligned}\right\} \text{Constraints on aggregate Club and Hotel allocations}
\end{aligned} \qquad (1)$$

Here, the shadow costs denoted by λ impose binding equality constraints for aggregating allocations within different market segments and Kuhn-Tucker shadow costs denoted by μ correspond to non-negative inequality constraints that may or may not bind on the final allocation.

Given the optimization problem (1) solves for a Pareto efficient allocation of licenses, the second welfare theorem states that this allocation can be supported as the competitive equilibrium of a market mechanism with associated prices. Those prices are approximated by the shadow costs for the binding constraints from the optimization problem. In practice, the constraint on aggregate Club allocations was not binding whereas the constraint for aggregate Hotel allocations did bind. Further, not all areas' allocation constraints were binding, so these constraints only affected the prices paid for licenses within those areas facing binding constraints. Accounting for these binding constraints, the relationship between the shadow costs and approximate prices for different types of licenses is summarized in Table 1.

In section 4 we demonstrate the second welfare theorem's application to the gaming auction problem. First, we define the market segments for licenses available to each type of bidder and characterize the derived demand for and supply of each type of license. The intersection of derived demand and supply in each market segment represents the market clearing price for that market, corresponding to the shadow costs from the constrained optimization problem and establishing partial equilibrium in each market segment.

Table 1: Lagrangian Shadow Costs and Approximate Prices

<table>
<tr><th>Type of License</th><th>Binding Constraints</th><th>Shadow Costs</th><th>Approximate Price</th></tr>
<tr><td>Club License in Unconstrained Area</td><td>Total Allocation</td><td>μ_O</td><td>μ_O</td></tr>
<tr><td rowspan="2">Hotel License in Unconstrained Area</td><td>Total Allocation</td><td>μ_O</td><td rowspan="2">$\mu_O + \mu_H$</td></tr>
<tr><td>Hotel Allocation</td><td>μ_H</td></tr>
<tr><td rowspan="2">Club License in Constrained Area a</td><td>Total Allocation</td><td>μ_O</td><td rowspan="2">$\mu_O + \mu_a$</td></tr>
<tr><td>Area Allocation</td><td>μ_a</td></tr>
<tr><td rowspan="3">Hotel License in Constrained Area a</td><td>Total Allocation</td><td>μ_O</td><td rowspan="3">$\mu_O + \mu_H + \mu_a$</td></tr>
<tr><td>Hotel Allocation</td><td>μ_H</td></tr>
<tr><td>Area Allocation</td><td>μ_a</td></tr>
</table>

Section 3: Elicitation to Determine Allocations and Prices

In practice, bidders report valuation schedules through a bidding mechanism. We defer a discussion of bidders' incentives and the conditions under which they can revise bids based on provisional allocations until section 5. For now, we describe the submitted bid schedules under the simplifying assumption that reported bid schedules reveal bidders' true valuations to illustrate properties of the allocation.

3.1 Reported Bid Schedules and Accumulated Bid Functions

Each establishment submits a bid schedule containing L_i entries specifying its quoted willingness to pay for each marginal good. The lists' entries are sorted by descending bid and the entry at the l^{th} level in the bid schedule is denoted $B_{il} = (b_{il}, x_{il})$. The bid price, b_{il}, reports the price the bidder is willing to pay and the quantity, x_{il}, reports the number of marginal units the bidder demands at that price in addition to the units they'd receive from any higher-priced bids. Bidder i's cumulative bid schedule, denoted $X_i(p)$, reports the total quantity of bids in the list with reported value weakly greater than p and can be computed by summing $X_i(p) = \sum_{l=1}^{L_i} x_{il} 1\{b_{il} \geq p\}$.

From the reported bid schedule, let $\hat{V}_i(x)$ denote bidder i's cumulative reported valuation for an allocation of x licenses. We calculate $\hat{V}_i(x)$ by summing the area under the bidder's reported bid function up to the quantity of x, characterizing the total value the bidder reportedly assigns to the allocation.[5] Since $\hat{V}_i(x)$ can be evaluated for any quantity of licenses, it can also be stated as a function of price evaluated at bidder i's cumulative bid schedule. Denoted $\hat{V}_i(X_i(p))$, this states the total valuation bidder i assigns to the number of licenses they would bid for if the price were p.

Table 2 provides a hypothetical example of an individual bid schedule and its translation into cumulative bids and reported valuations. Panel A presents a schedule with four entries at four different price points for an establishment that bids for up to 25 units if the price is less than 80. The Cumulative Bid Schedule in Panel B demonstrates how different prices translate into total quantity desired by that bidder at each price.

Table 2: Sample Individual and Cumulative Bid Schedules

Panel A: Reported Bid Schedule

List Entry [l]	Bid [b_{il}]	Bid Quantity [x_{il}]
1	100	5
2	95	5
3	90	10
4	80	5

Panel B: Cumulative Bid Schedule

Price [p]	Cumulative Bid [$X_i(p)$]	Cumulative Reported Value [$\hat{V}_i(X_i(p))$]
100	5	500
96	5	500
95	10	975
90	20	1,875
80	25	2,275

[5] The formula for $\hat{V}_i(x)$ is a bit convoluted, due to the discrete nature of bids, but can be calculated as:

$$\hat{V}_i(x) = \sum_{l=1}^{L_{il}} b_{il}\left[x_{il} 1\left\{ \sum_{j=1}^{l} x_{ij} < x \right\} + \left(x - \sum_{j=1}^{l-1} x_{ij} \right) 1\left\{ \sum_{j=1}^{l} x_{ij} \geq x \right\} 1\left\{ \sum_{j=1}^{l-1} x_{ij} < x \right\} \right]$$

3.2 Implemented Allocation Rule

The system approximates the continuous model presented in section 2.2 with the elicited valuations as reported in section 2.3. Measuring total welfare by the aggregated reported license valuations, $\hat{V}(X)=\sum_{i\in I}\hat{V}_i(x_i)$, and maintaining all the relevant constraints, the system determines the allocation by optimizing:

$$
\begin{aligned}
\hat{\mathcal{L}}(X) = {} & \hat{V}(X) \\
& \left.\begin{aligned}
&-\sum_{a\in A}\lambda_{aC}\left(x_{aC}-\sum_{i\in I}x_i(1-h_i)1\{a_i=a\}\right) \\
&-\sum_{a\in A}\lambda_{aH}\left(x_{aH}-\sum_{i\in I}x_i h_i 1\{a_i=a\}\right) \\
&-\sum_{a\in A}\lambda_a\left(x_a-x_{aC}-x_{aH}\right) \\
&-\sum_{a\in A}\mu_a\left(\overline{x}_a-x_a\right)
\end{aligned}\right\}\text{Constraints on area Club and Hotel allocations} \\
& \left.\begin{aligned}
&-\lambda_H\left(x_H-\sum_{a\in A}x_{aH}\right)-\lambda_C\left(x_C-\sum_{a\in A}x_{aC}\right) \\
&-\mu_H\left(\overline{x}_H-x_H\right)-\mu_C\left(\overline{x}_C-x_C\right) \\
&-\mu_O\left(\overline{x}-x_H-x_C\right)
\end{aligned}\right\}\text{Constraints on aggregate Club and Hotel allocations}
\end{aligned}
\qquad (2)
$$

The discrete nature of the problem introduces a number of complexities in solving the optimization problem from equation (2). The complexities are well known features of integer programming optimization, requiring tie-breaking rules, non-uniqueness of shadow costs, and potential for multiple solutions due to overlapping constraints. Tied bids are resolved through a first come, first serve rule. Since all bids are time-stamped, if multiple bids are submitted at the market quoted price, then allocations are made to the bids according to their arrival time. The market clearing prices need not be unique if the quantity demanded at a price exactly equals the supply to that market and thus multiple prices can clear the market. This was addressed by adding a very small quantity to every bid so the quantity demanded at a price is always slightly above the integer parts. Finally, though multiple constraints could bind and thus create multiple price solutions, such multiplicity could only arise from relationships between metropolitan constraints and area constraints. The problem never surfaced because the metropolitan constraint was never binding.

Given the adjustments to the optimization problem necessary to address these practical considerations, the shadow costs from the optimization problem only approximate the shadow costs from the continuous Lagrangian in equation (1). As we will see in section 4, these approximations do not induce disequilibrium in any individual market's allocation.

3.3 Preview of Dynamic Auction Features and Bidding Revisions

The system arrived at its final allocation after progressing through a series of 63 bidding rounds. Section 5 presents a detailed discussion of bidding dynamics and general equilibrium convergence. Each round begins with establishments submitting provisional bid functions. The auction algorithm computes the allocation and prices based on these bid functions. These provisional prices and allocations are then announced at the conclusion of the round. Thus bidders observe the quantity of entitlements that they would purchase, and the price they would pay per unit, if the auction were to stop in that round.

Given this information, bidders are aware of the prices that revised bids need to meet or beat if they wish to obtain additional licenses. Before beginning the next round, bidders can take advantage of this information and revise their submitted bid functions, subject to the restriction that they increase their original bid by at least a minimal increment. This restriction induces an ascending auction format and excluded bid pricing format as bidding progresses from round to round.

The system initiates an ending process based on the numbers of significant revisions (according to rules from testbed experiments) in individually submitted bid schedules and related patterns of market price changes. At this point, bidders are notified about the number of rounds the market will remain open terminating with the announcement that the market will close in the subsequent round and given a final opportunity to revise their bid schedules. If there are still no significant revisions in this next round, the auction closes. After this last round, the bidders pay the announced price for their market for each entitlement awarded.

3.4 Testbedding Designs to Determine Parameters

The mechanism design exercise began with a theoretical sketch of an auction where: (i) each bidder preferences were limited to a single establishment; (ii) agents submit "truthful" demand functions; (iii) the auction winner is chosen by maximizing the revealed value of the allocation; and (iv) policy limits regarding multiple markets exist as constraints. The theoretical sketch did not address behavior and implementation, even at an experimental, testbed level, but identified the analytical principles guiding the mechanism and technical challenges that to be addressed in that implementation. Complex auction experiments regarding call markets (Plott and Pogorelskiy, 2017), power grids (Chao and Plott, 2009) and combinatorial auctions (Lee, Maron, and Plott, 2014) offered prototypes that address important aspects of the theory necessary for implementation. However, other experiments with pure tatonnement (Plott, 2001) and on Natural gas pipelines (Plott, 1988) served as a warning about the incomplete nature of theory as a model of actual behavior. A key behavioral feature, demand revelation at the margin, had also been identified in an unpublished working paper on fuel efficiency under the CAFE constraint (Katz and Plott, 2009).

Experimental work recommended an auction based on dynamic equilibration and convergence as opposed to a one shot, sealed bid computation. Early experimentation with auction architectures focused on continuous processes using tatonnement type process or double auctions as opposed to sealed bid system. The scale of the gaming machines auction and nature of the multi-unit demand for licenses required analyzing functions (inverse demand functions) as opposed to separate bids on individual units. Competitive theory as applied to smooth demand functions identified relationships among bids, prices (as Lagrangian multipliers), equilibrium (as a competitive equilibrium), allocations and efficiency. The actual auction required solving an integer-constrained, linear program for the allocation problem. The size of the problem and ability of bidders to deal with information flow was best accommodated by a round structure as opposed to continuous bidding that had proved successful in both experiments and the field.

3.4.1 Testbedding Designs at Scale

The scale of the design problem derives from the size of markets, the number of markets, the number of units, and number of bidders. The testbedding exercise focused on establishing the economic performance and technical control of the system. The scaling technique relied on homogeneity properties of economies. If agents have linear demand functions, increases in the number of demanders of each type, accompanied by appropriate supply increases, will leave prices and individual allocations of a given type unchanged. The experimental scaling started with only two areas (small and large), a fixed number of hotels and clubs in each and a fixed number of entitlements to be equally split between clubs and hotels. More complex designs replicated this module, increasing the number of markets, bidders, and licenses to be allocated. This scaling method tested key aspects of convergence, efficiency, and computation time in relation to the size of the economic problem. It did so without a need to undertake complex computational challenges that can exist with large numbers of markets.

Two performance measures were useful tools to refine the rules of the auction. The first was market efficiency measured as consumer surplus as developed by Plott and Smith (1978). This measure is the sum of observed willingness to pay divided by the maximum sum of willingness to pay given experimentally induced preferences. The second was speed of convergence measured in terms of number of rounds required for equilibration. The scale testing experiments started with 8 participants, 4 regions and 8 markets.[6] Participants and numbers of markets were scaled up such that prices and allocation per type remained approximately the same to study process characteristics in response to scale. Over 40 different experiments were performed and often repeated to explore any problems exposed. The

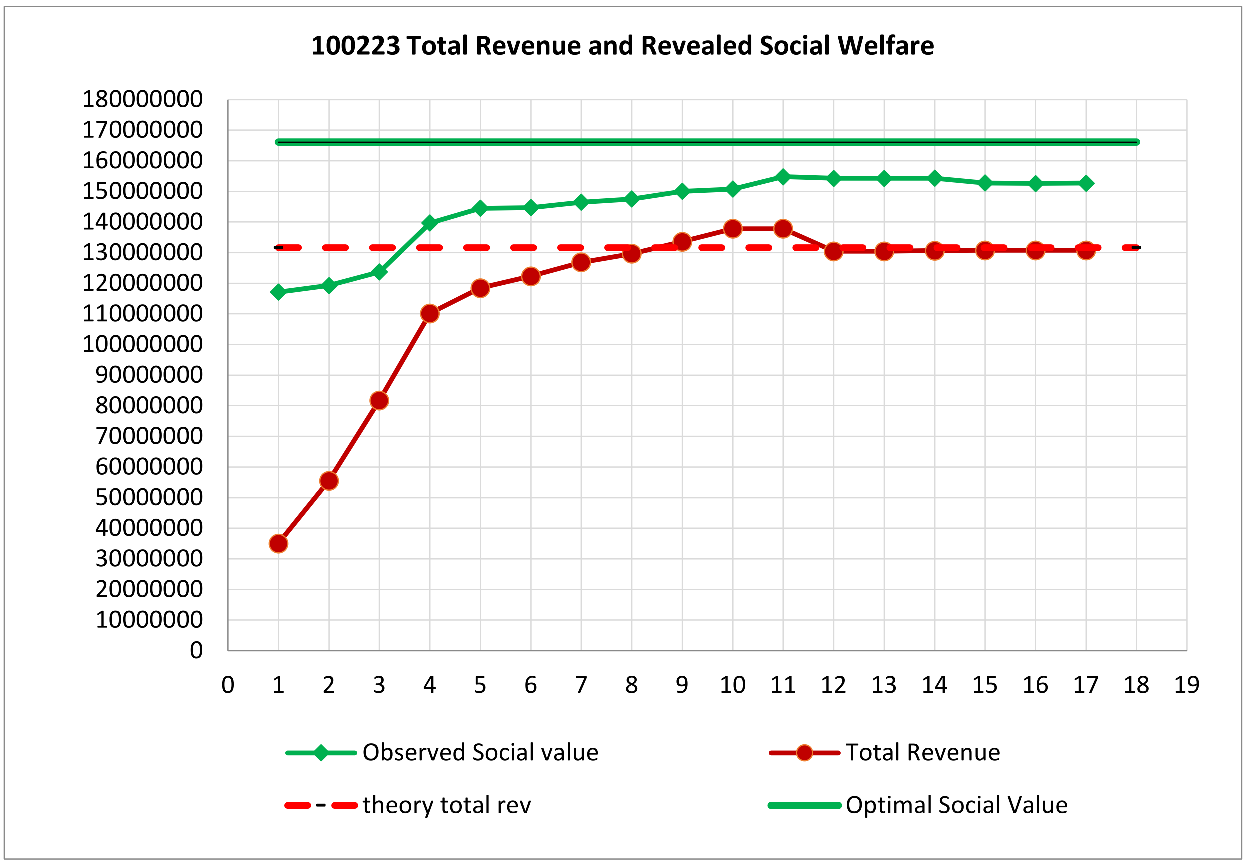
[6] In some of the early experiments, each market contained only one bidder. Competition emerged from the interconnection of markets as any excess supply in a market could be absorbed by other markets.

largest testbed featuring human subject participants operated with 50 markets and 160 participants at a cost of $8,866. Larger scales were simulated with multiple computers programmed to place bids to test network configurations, processing speeds, and computation reliability and speed.

3.4.2 Testbedding Parameters for the Mechanism

Experiments revealed the ability of the mechanism to coordinate convergence over multiple markets, including derived demand and prices. Allocations and prices typically ended near the predictions of the general competitive equilibrium and thus efficiency tended to be in the high 90% levels and often near 100%. Such high performance occurred at all tested levels of scale. Figure 1 illustrates the rapid rise of total auction revenue and social value (gains from trade) in the first few rounds for one experiment.[7] When the auction closed in the 17th round, revenue almost exactly matched the theoretical equilibrium, and over 90 percent of the optimal social value was realized.

Figure 1: Total Revenue and Social Welfare for an Example Experiment



[7] This experiment focused on five areas. Each area had a club market and a hotel market. Thus there were ten markets and thirty subjects. Each subject had a linear induced demand.

While each experiment examined multiple dimensions of performance, a narrow focus was on two areas. The first related to real time control of price movement and ending the auction. Previous experimental work revealed that bidding incentives and stopping rules are important for performance. The second broad area included performance, efficiency and reliability in both a software and behavioral sense.

The timing of the auction rounds needed adjustments to account for the reaction speed of bidders. Bidder behavior reflected their own information and preferences as well as a second, strategic, dimension affecting the degree to which bidders should "reveal" the willingness to pay by increasing bids. An incremental requirement defining the minimal allowable increase had an obvious role ensuring revisions were economically meaningful. We adopted a two-clock methodology for controlling bidder behavior.[8] Experiments suggested the use of discrete rounds as opposed to continuous time to control information reporting and allow bidders time to process price and allocation information. How scale, in term of numbers of market and bidders involved, would affect mechanism performance involved unknowns that were revealed through the testbed process.

Requiring only a single bid revision for auction continuation is not practical because various levels of randomness in bidding and bidding timing are always present. The practical question becomes "how many" bids or price changes in a round does it take to justify keeping markets open for additional rounds. Testing in experiments with different controls led to the number of bidders that attempted to increase their holdings as the controlling measurement for the first clock and number of markets that changed prices as the controlling measure for the second clock. Time was measured in number of rounds required before a change in these thresholds. Changes were well publicized. New bid increment requirements were announced as a percentage over existing prices and these were enforced beginning in a specified round in the future. Specific hypotheses about responses to these controls were impossible to test given the size, time, and expense of experiments. Nevertheless, the experiments provided substantial experience with how the auction would respond to the chosen parameters.

Adjustment speed depended on the bidding response to price increments. The responses were the primary source of information about the likely auction ending time. How long the auction might take was important given the government's decision to limit it to one day. Experiments demonstrated that bidding followed a principle of revelation at the margin.

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[8] In continuous time actions one clock counts down in seconds and resets with the submission of a new bid in any market. If no bid appears in any market, the clock counts down to zero and the auction ends. A second clock is employed in auctions with complex bids, such as bid functions, because new bids need not result in price changes and can become cheap talk that simply keeps the auction open. Such possibilities are avoided by the use of a second clock that counts down and resets if a bid results in new winners, thus exerting pressure to place bids that cause prices to evolve.

Announced prices were accompanied by the increment requirement. All new bids or changed bids must be no less than the existing price plus increments. Unchanged bids remain in the system. The upcoming price is not known and price could remain unchanged. New bids are automatically integrated with the bidder's existing bids to form a revised bid function. The new bid function is a type of "revealed demand" but it is not fully revealing. The revealed demand function always falls short of the limit values (demand prices) of infra-marginal units. Since the actual demand functions are known in testbed experiments the size of this "shortfall" can be observed directly.

While the revealed demand curve falls under the actual demand curve an important element – demand at the margin – is accurately revealed. The experiments indicated that the quantity demanded at the announced price is always very close to the quantity demanded according to the induced preferences. Given the rules, the bidder can adjust the bid price to assure the purchase of the marginal unit given the announced price and do so without directly influencing that price. To express a preference for an additional unit at the stated price the subject merely needs to express a willingness to pay for it by tendering a bid price for the unit above the stated market price. Thus, the value of the marginal demand is revealed. The demand function becomes traced out as price moves up following the required bid increments. In particular, the slope is useful information revealing the state of demand relative to prices and thus when the auction is near a competitive equilibrium. This key feature is discussed in more detail in Section 5 and illustrated in Figures 5 and 6.

3.4.3 Testbedding Parameters for the Mechanism

The experiments were the only source of information about bidder behaviors and perceptions. Our assessment of instructions, bid submission screens, bidder feedback, tools for bidder expression, and other subtle variables related to bidder perceptions and reaction came from experiments. The experiments and experimental subjects shaped all instructions and the technical aspects of the human-to-auction interface. Probity issues placed governmental constraints that limited how the auction designers could interface with individual bidders. The government prohibited all interactions with potential bidders or anyone else that might interact with bidders, due to fears that such interactions might create unfair advantages among bidders. Consequently, computer screens, instructions, explanations of bid functions versus bids, increment requirements, stopping rules, price determination, the time taken for individual decisions and many other variables related to preconceptions, perceptions and skills of the bidder pool could not be studied directly. The designers had no contact with actual bidders. The designer's understanding of these variables was acquired entirely through laboratory experiments using college students as subjects.

Other important issues such as how to deal with typos or slow computational speeds were studied through experiments. Concerns about hackers, power reliability, and local computers and network reliability were based on theory and experiments with limited opportunity to test the actual hardware system that was used.

Section 4: Partial Equilibrium Properties of the Final Allocation

This section relates the system's algorithm for determining the allocation and prices to a competitive model of demand under inelastic supply with segmented markets. In doing so, we describe what might be modeled as partial equilibrium properties of the final allocations in each market segment. We discuss each segment and its partial equilibrium model separately. Section 5 presents more details of the bid revision process and dynamics that determine total supply across each of the markets, including bidders' incentives to truthfully reveal their demand through these revisions.

We begin by defining the different market segments based on the prices paid for different licenses and aggregate the derived demand for establishments competing to obtain these licenses based on firms' reported bid schedules. Within each of the market segments, the derived supply is price-inelastic at a fixed quantity.[9] The price at which derived demand equals derived supply in each market is approximated by the price premia for different types of licenses from the constrained optimization problem determining the allocation. As such, the final allocation and prices in each market segment are consistent with an ex-post partial equilibrium given bidders' unwillingness to submit revisions to their reported bid schedules.

We identify the different market segments by the inequality constraints in the optimization problem that bind on any given allocation. Throughout our analysis, we assume three stylized facts that applied to all allocations in the system that allow us to identify these segments. First, we partition the set of areas $A = A_C \cup A_U$ into constrained areas A_C (where $x_a = \overline{x}_a, \forall a \in A_C$) and unconstrained areas A_U (where $x_a < \overline{x}_a, \forall a \in A_U$). Second, hotel establishments were allocated the maximum number of licenses available based on their venue type ($x_H = \overline{x}_H$) while club establishments' allocation did not meet this maximum ($x_C < \overline{x}_C$). Third, all licenses available to the system were allocated (i.e., $\overline{x} = x_H + x_C$). These assumptions give rise to two large market segments for unconstrained clubs and unconstrained hotels and a number of smaller market segments for each constrained area in A_C commanding a local price premium.

4.1 Partial Equilibrium in the Market for Unconstrained Clubs

[9] The derived quantity supplied may depend on allocations to other market segments, though this feature of the market system doesn't impact the partial equilibrium analysis in this section. We return to discuss of the general equilibrium properties of the market system below when analyzing stability of excess demand functions and a model of convergence to an equilibrium.

Consider the market for club licenses in unconstrained areas under an allocation in which $x_C < \bar{x}_C$. Bidders in these markets compete with each other for the pool of licenses that are not allocated to hotels or to any of the constrained areas. Collectively, the aggregated bid schedules for bidders in these markets identify the *Derived Demand* for Unconstrained Clubs, which can be calculated as:

$$D_{UC}(p) = \sum_{a \in A_U} \sum_{i \in I} X_i(p)(1-h_i)1\{a_i = a\}.$$

The supply available to these bidders is determined after all constrained area markets for both clubs and hotels, and the unconstrained hotel market have already cleared:

$$S^t_{UC}(p) = \bar{x} - x_H - \sum_{a \in A_C} x_{aC}.$$

Finally, define the "Club Base Price," p^*_{UC}, as the derived market clearing price where $D_{UC}(p^*_{UC}) = S_{UC}(p^*_{UC})$.

Figure 2: Derived Demand and Supply for Unconstrained Clubs

Panel A: Full Demand and Supply Panel B: Equilibrium Detail

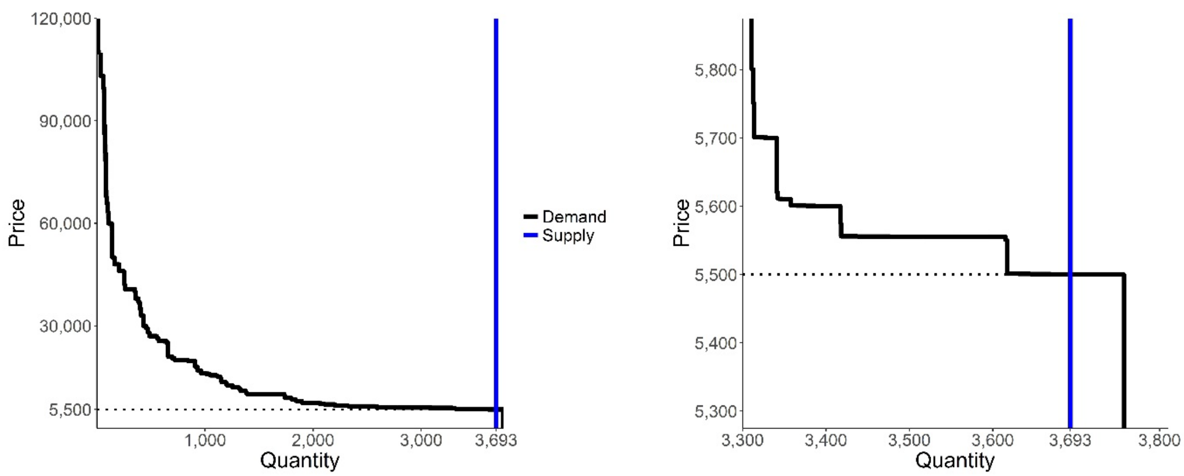


Figure 2 presents the derived demand and supply for unconstrained clubs in the final round of the system's operation. The residual supply available to this market consisted of 3,693 units, which matched Derived Demand to set the Club Base Price of \$5,500. Due to a large mass of demand at \$5,500, the quantity demanded at this price exceeded the available supply. The proportion of demand met at this price was determined by priority based on when establishments submitted their bids. The mass at this price point has two important implications for the system. First, any establishment that did not receive an allocation at the \$5,500 price could have increased their bid to receive additional licenses with no impact on price. Given the quantity supplied at this margin, an establishment could have obtained up to 61 additional licenses without having any impact on price, a number of licenses that was greater than the number acquired by any establishment. Second, the unmet demand of 64 units at the external margin suggests that no establishment would have any power to reduce prices by lowering their bids while still receiving the same allocation. If any bidder were to reduce their stated value for licenses allocated at this external margin, they would lose those licenses to the unmet demand.

Finally, we connect the price for this market segment to the optimization problem (1). Note that the allocation of all other license types are at their constrained maxima, whether due to individual area constraints or the aggregate hotel maximum license constraint. Consequently, any additional licenses made available to the market in total (i.e., an increase in $\overline{x}$) would be sold in the unconstrained club market at price p^*_{UC}. As the potential increase in the value function from relaxing the total license quantity constraint, this price must then equal the shadow cost of the constraint so that $p^*_{UC} = \mu_O = 5,500$.

This analysis demonstrates that the market for unconstrained club licenses cleared in a classical sense. Bidders for unconstrained club licenses could unilaterally revise their bid schedules to increase their provisional allocation without changing the prices they would pay for the allocated licenses. Bidders could cancel bids that were not provisional winners and avoid the possibility that the bids would be filled on subsequent rounds. That bidders chose not to take advantage of such revision opportunities demonstrates the allocation of licenses among bidders for unconstrained club licenses represents an ex-post equilibrium.

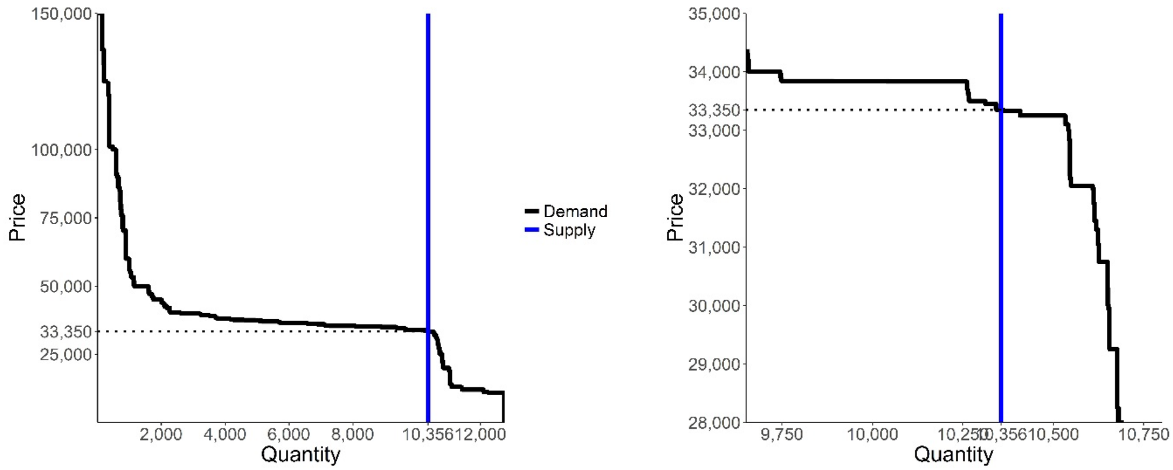
4.2 Partial Equilibrium in the Market for Unconstrained Hotels

Next consider the market for licenses for hotel establishments competing in unconstrained regions. Similar to the market for unconstrained clubs, these bidders compete solely with each other for the pool of hotel licenses that are not allocated to any of the constrained areas. Define the t^{th} round Derived Demand and Supply, respectively, for Unconstrained Hotels as $D_{UH}(p) = \sum_{a \in A_U} \sum_{i \in I} D_i(p) h_i 1\{a_i = a\}$, and, $S_{UH}(p) = \overline{x}_H - \sum_{a \in A_C} x_{aH}$. The "Hotel Base Price" represents the market clearing price, p^*_{UH}, that balances derived demand and supply so that $D_{UH}(p^*_{UH}) = S_{UH}(p^*_{UH})$.

Figure 3 presents the derived demand and supply for unconstrained hotels in the final round of the system's operation. The 10,356 units of supply available in this market matched Derived Demand to set the Hotel Base Price of $33,350. Derived demand for hotel licenses appears relatively elastic compared to demand for unconstrained club licenses, likely as a consequence of greater heterogeneity in values for establishments in this market segment. Still, market power and market impact for bidders is quite limited, with fourteen units allocated out of twenty demanded at the Hotel Base Price. Consequently, a bidder in the unconstrained hotel market would be able to obtain an additional fourteen licenses by stating a higher willingness to pay without impacting their actual price paid. Bidders could also reduce their stated willingness to pay for the non provisional winning bids and thus avoid acquiring units in subsequent rounds.

Figure 3: Derived Demand and Supply for Unconstrained Hotels

Panel A: Full Demand and Supply Panel B: Equilibrium Detail



From the optimization problem (1), the Hotel Base Price reflects the marginal revenue available from selling one additional license to a hotel, effectively increasing $\overline{x}_H$ by one unit. However, holding $\overline{x}$ constant means this unconstrained hotel license must come from the supply of unconstrained club licenses. Combined, the Hotel Base Price equals the shadow cost of the constraint on the supply of hotel licenses plus the shadow constraint on the total supply of licenses, i.e., $p^*_{UH} = \mu_O + \mu_H = p^*_{UC} + \mu_H$. We refer to the margin between the Base Hotel Cost and Base Club Cost, $\mu_H = 27,850$, as the Hotel Price Premium.

As is the case with the market for unconstrained club licenses, the analysis demonstrates that the market for unconstrained hotel licenses cleared in a classical sense. Bidders in this market retain the unilateral ability to revise their bid schedules and alter their provisional allocation without changing the prices paid. Consequently, bidders' revealed unwillingness to make such revisions demonstrates the ex-post equilibrium nature of the allocation among bidders for unconstrained hotel licenses.

4.3 Partial Equilibrium in the Markets for Licenses in Constrained Areas

In areas where the quantity of licenses is constrained, Clubs and Hotels compete with each other to determine the Area Price Premium. The derived demand for licenses in these areas aggregates the demand schedules for bidders in the area in excess of the base price determined by the unconstrained markets for each venue type:

$$D_a(p) = \sum_{i \in I} \left[X_i \left(p + p_{UH}^* \right) h_i + X_i \left(p + p_{UC}^* \right) (1 - h_i) \right] 1\{a_i = a\}.$$

Impounding the venue base price into the demand schedule translates each establishment's bid in terms of the premium realized by not allocating the license to a bidder in an unconstrained market. In constrained markets, the inelastic derived supply is fixed at the maximum constraint so $S_a(p) = \overline{x}_a$. The "Area Price Premium," denoted p_a^*, represents the price that clears the market, so that $D_a(p_a^*) = S_a(p_a^*)$.

Figure 4: Derived Demand and Supply for Constrained Area 110

Panel A: Full Demand and Supply Panel B: Equilibrium Detail

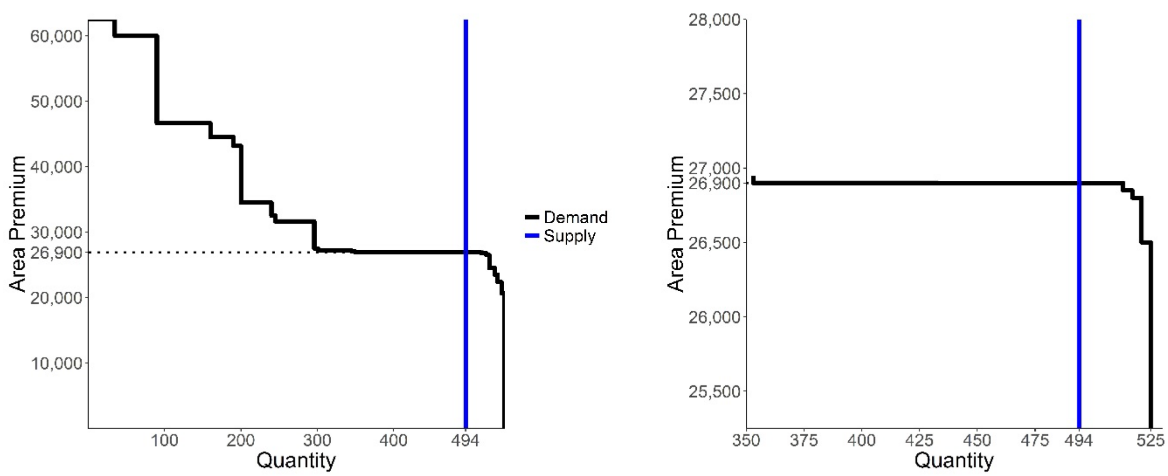


Figure 4 presents the derived demand and supply for the final allocation in area number 05, with the schedules for all other areas appearing in Appendix A. The total allocation of 494 licenses for this area is much smaller than for the unconstrained markets, with the market clearing at an area price premium of $26,900. Total demand at this external margin was 160 units, leaving 19 units of unmet demand at this price. As in the unconstrained markets, this atom of demand demonstrates bidders' ability to increase their allocations by revising their bid schedules without impacting prices and their unwillingness to cancel non-provisional winners to avoid acquiring additional units at the stated market price.

Key Result 4.0: The prices and allocations for each license type is determined by the bid schedules submitted for bidders seeking to acquire that type of license. The solution to the Victoria Gaming Auction satisfies classical market clearing conditions balancing derived demand

and derived supply within each market segment. This consistency suggests the market for each license type achieves a partial equilibrium for allocating its inelastic supply.

Section 5: Bidding Dynamics, Excess Demand Revelation, and Equilibrium Convergence

We now present the auction's dynamic features, driven by shifting bid functions that can be modified, supplemented, or cancelled across rounds. At the conclusion of each bidding round, provisional prices are announced for each market so bidders are aware of the prices they need to meet or beat to obtain a different allocation of entitlements. To make the price determination more transparent we adopted last-accepted bid rules for the uniform price, rather than first-rejected bid rules. Although these uniform price rules do not make value revelation incentive compatible, as described earlier they nevertheless encourage value revelation on the margin as the market price rises through successive bidding rounds. Identifying the demand revealed at the margin as a measure of excess demand for licenses suggests prices adjusted following a tatonnemont-like process. Using the rich bidding data, we estimate the parameters of this price adjustment process to empirically verify it satisfies well-known stability conditions for general equilibrium. The theoretical and empirically verified properties of the system demonstrates the final allocation achieved general equilibrium conditions to represent an efficient allocation across markets.

5.1 Demand Revelation and Incentives at the Margin

To track revisions in bid schedules across rounds, superscript bids, prices, and demand calculations with their associated round. To illustrate, $B_{il}^{t}=\left(b_{il}^{t},x_{il}^{t}\right)$ represents the l^{th} entry from the i^{th} establishment's bid schedule submitted in the t^{th} round of the mechanism with associated cumulative bid schedule $X_i^t\left(p\right)$. Similarly, let x_i^t denote the t^{th} round's provisional allocation to establishment i, with p_{aH}^{t} and p_{aC}^{t} identifying the market clearing prices for this allocation. Given the uniform pricing rule, these prices identify the point in the demand schedule at which a bidder's incentives for truthful demand have a binding property.[10] For bid schedule entries with prices above or below this margin, a bidder could respectively inflate or deflate their stated willingness to pay without changing their allocation or the payment required to receive that allocation.

Round t opens after announcement of the provisional prices and quantities allocated based on the bid functions submitted by all bidders in round t-1. As the solution to the

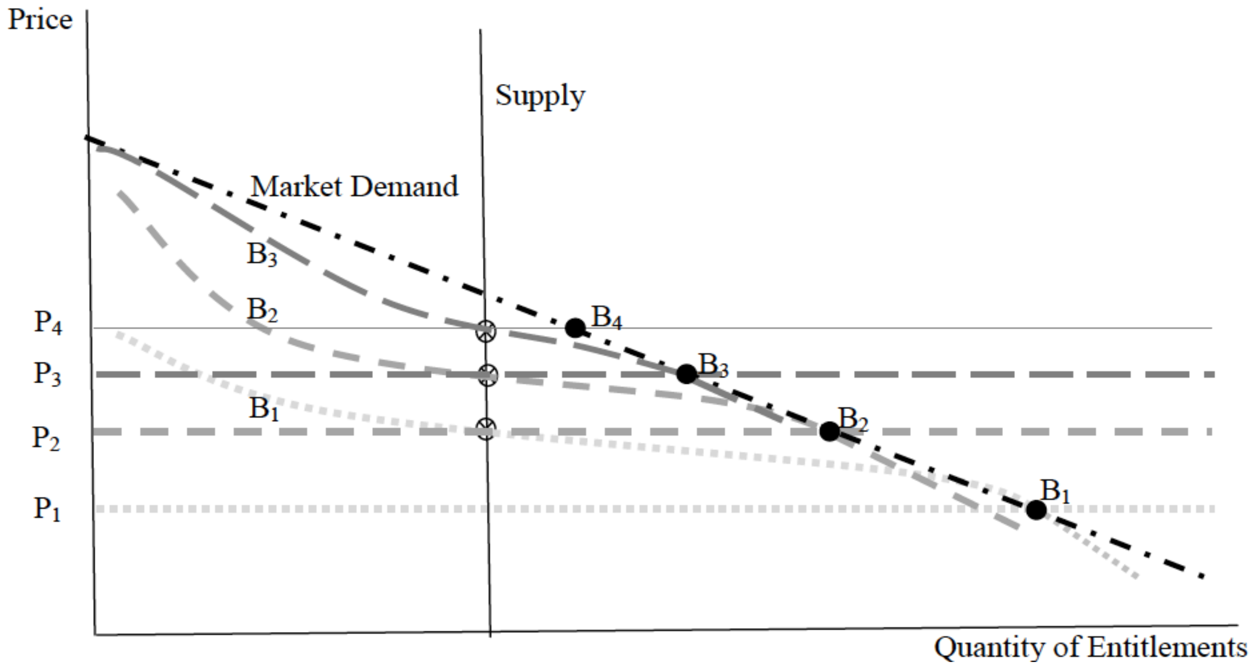
[10] As demonstrated in Section 4, no individual bidder has the power to influence prices, so truthfully reporting their demand ensures they obtain exactly the quantity they want regardless of the final price. As advice to bidders, market designers suggested taking this approach when submitting bids. While some bidders followed this suggestion, the pilot experiments and testbed studies demonstrated the vast majority of bidders responded to announced prices and provisional allocations in a marginal manner.

optimization problem, these provisional allocations and prices each satisfy the partial equilibrium conditions balancing derived supply and reported demand demonstrated in section 4. Bidders respond by submitting round t's bid schedules. If the bidder is satisfied with their t-1 round allocation at the t-1 round's ending price, they would have no incentive to revise their reported bid schedule. If all bidders are satisfied with their t-1 round allocation at the t-1 round's ending price, then no bidders would revise their reported bid schedules and the market would close due to inactivity, determining the final allocations and prices.[11] Given their potential as final auction outcomes, the provisional allocations and prices offer incentives for further revelation due to threat of closing.

However, if a bidder wanted to increase their t-1 round allocation, they could do so by increasing the bid price for some entries in their schedule to be above the t-1 round ending price. Bid revisions arise when bidders decide they want a greater allocation at the announced price, with the system prompting a bidder to consider whether they want more licenses at the announced price. The system incentivizes bidders to report the quantity they wish to buying at (or slightly above) the publicly announced prices for the market. At each newly announced price, bidders reveal the maximum quantity they want at that price, a process we refer to as "demand revelation at the margin."

[11] When there was an insufficient amount of bidding activity in a round, the auctioneer would publicly announce that the auction will close if there is insufficient activity in one more round of bidding. If that tentatively "last" round of bidding featured significant revision activity, the auction would again proceed until revision activity ceased again. The auctioneer would then repeat their announcement and the process would continue until there is insufficient revision activity in that "last" round of bidding. In practice, the auctioneer announced only two tentative ends to the bidding process, with the second corresponding to the close of the auction.

Figure 5: Illustrating Dynamics of Demand Revelation



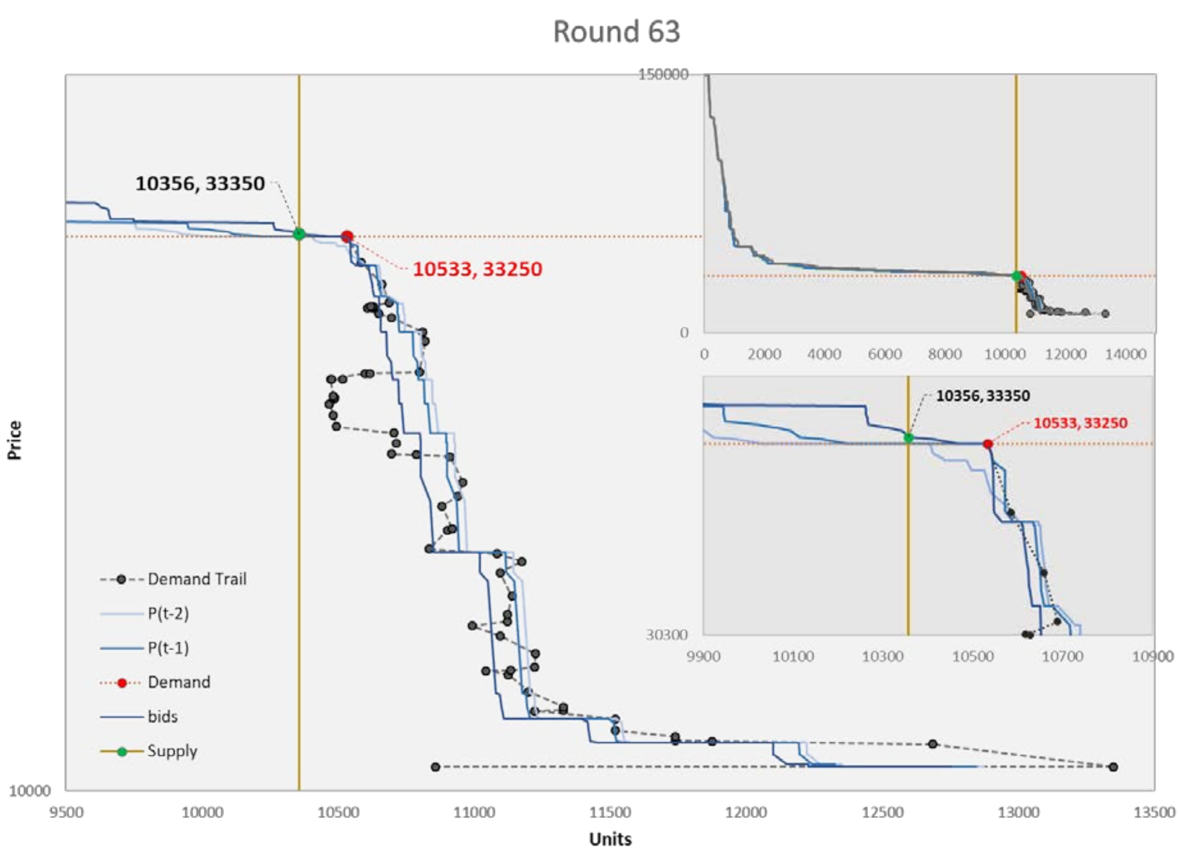
Through demand revelation at the margin, rising prices across auction rounds “trace out” points on the market’s aggregate demand curve as illustrated in Figure 5. The market opens with an initially announced price of P_1, leading bidders to reveal (approximately) the actual quantity demanded at this price in the subsequent round, but with potential under-revelation of demand at higher prices as illustrated in aggregated bid function B_1. The reported bids generating bid function B_1 lead to an announced price of P_2. The round 2 bid revisions generate a new revealed demand curve, with the point B_2 indicating the true quantity desired at this price. The reported bids generating aggregated schedule B_2 lead to a market clearing price of P_3, which then becomes the operative point of demand revelation for the round 3 bid revisions. As rounds progress and prices continue to rise, demand revelation at the margin will trace out more points very close to the true demand curve until the system converges.

Figure 6 contains the time path of revealed demand at the margin for the unconstrained hotels, discussed above in Section 4.2. The total number of units allocated to constrained hotels changes very little over periods so the supply of entitlements to constrained hotels remains relatively constant (as illustrated in the figure). Shown for each period is the aggregate demand revealed at the market price of for unconstrained hotels. Total demand revealed at the margin appears somewhat inelastic and with the difference between total excess demand at the margin and supply slowly shrinking to zero where the auction terminates. The panels at the upper right show the details in terms of units of excess demand at the end of the auction [quantity demanded at the margin – supply = 10509 - 10332] and also the entire demand curve as revealed at end of the auction. We formalize these measures of excess demand for more detailed analysis in the next section.

This multi-round bidding process thus induces a price adjustment process analogous to tatonnement, but with the complication of potential uncertainty in how newly submitted bid functions collectively lead to further price adjustments. As prices converge over time, bidders can eventually develop confidence that they will become provisional winners for quantities bid at prices that exceed previous round provisional prices. Such confidence is especially warranted for bidders representing Clubs and Hotels in unconstrained geographic areas, where individual bidders are very small relative to the market.

KEY RESULT 5.1: New Principle: "Revealed Demand at the Margin". Bidders' incentives only bind at the margin, causing their demand at interim prices to be truthfully revealed.

Figure 6: Marginal Demand Revelation for Unconstrained Hotel Licenses



5.2 Measuring Excess Demand Revealed at the Margin

We now empirically evaluate the extent to which demand is revealed at the margin as the system progresses through rounds. To begin, define the round *t* Revealed Demand at any price p as the difference between the round *t* demand and the round *t*-1 demand:

$$\Delta X_i^t(p) = X_i^t(p) - X_i^{t-1}(p) \tag{3}$$

Note that, if a bidder submits an identical bid schedule in two subsequent rounds, $\Delta X_i^t(p)$ will be zero for all values of p.

Clearly, both the Demand and Revealed Demand measures can be aggregated to the market (area-venue) level by summing over bidders within each market. Define the aggregated (market-level) demand schedules as:

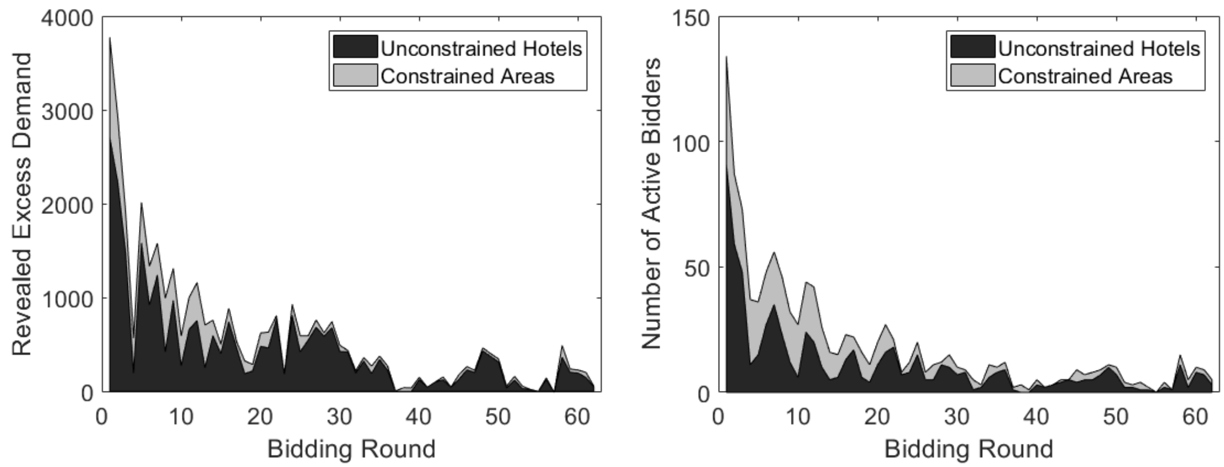
$$X_{aH}^t(p) = \sum_{i \in I} X_i^t(p) h_i 1\{a_i = a\}\text{, and, } X_{aC}^t(p) = \sum_{i \in I} X_i^t(p)(1-h_i)1\{a_i = a\}$$

$$\Delta X_{aH}^t(p) = \sum_{i \in I} \Delta X_i^t(p) h_i 1\{a_i = a\}\text{, and, } \Delta X_{aC}^t(p) = \sum_{i \in I} \Delta X_i^t(p)(1-h_i)1\{a_i = a\}$$

These measures characterize the aggregate revealed demand in a market as rounds of the mechanism progress, with the higher-level aggregates characterizing total Hotel, Club, and system-wide revealed demand for licenses at any given price.

Given binding incentives at the market clearing price based on the last round's submitted bid schedules, this "opening" price provides the most relevant measure of revealed demand. As the price bidders would have to exceed in order to obtain additional units relative to their provisional allocation, this price sets bidders' expectations for what bid values are likely to be awarded additional licenses. We adopt this measure, $\Delta X_i^t\left(p_i^{t-1}\right)$, as the empirical measure of Revealed Excess Demand at the Margin.

Figure 7 presents the time series of the total Revealed Excess Demand at the Margin from round 2 forward, disaggregated by the type of venue revealing demand. Panel A presents the number of units of revealed excess demand while Panel B reports the number of bidders whose changes in bid schedules revealed demand. As is apparent from the graph, all revealed excess demand is generated by revisions for unconstrained Hotels and bidders in constrained areas, with all revealed demand from Clubs associated with constrained areas. Turning toward the dynamics of aggregate Revealed Excess Demand at the Margin, note the downward trend and convergence toward zero in the later rounds of the market after a few minor bumps in later rounds.



Panel A: Revealed Demand Quantity Panel B: Number of Active Bidders

Figure 7: Revealed Excess Demand at the Margin

The revealed excess demand in the last round of bid revisions is attributed to the activity of a small number of bidders in only three markets for licenses in constrained areas. The isolated nature of these revelations suggest the system had largely converged by this point. Since these markets' allocations met the constrained maximum for these areas, any additional demand revelation would do nothing to change either the area-level allocation or the allocations in unconstrained markets. Rather, any changes to the allocation would only affect the allocation of licenses among those establishments operating within these three markets along with their associated prices.

KEY RESULT 5.2: Revealed Excess Demand at the margin is diminished and eliminated through the Victoria Gaming Auction.

5.3 Gross Substitutes, Stability, and Equilibrium Convergence

We now argue that, by satisfying and subsequently reducing revealed excess demand at the margin, the system guides the allocation to a general equilibrium across market segments. First, we demonstrate that the licenses in different markets represent gross substitutes. Second, the evolution of allocations as bidders revise their bid schedules corresponds to adjustments in prices that reduces excess demand. Combined, these suggest the price adjustment mechanism follows the principles of tatonnemont. Since tatonnemont processes converge to the unique, stable, general equilibrium for gross substitutes, we conclude that the market system's final allocation matches that of a general equilibrium.

From a theoretical perspective, the gross substitute property, i.e., that excess demand weakly increases in response to an increase in the price for another good, is a sufficient condition for stability of a general equilibrium system.[12] To establish the gross substitute property for licenses in different markets, consider the response of derived supply and demand for licenses in market *i* as the price for licenses in market *j* increases. On the supply side, the price increase in market *j* could increase the number of licenses allocated to market *j*, weakly reducing the number of licenses available in market *i*.

On the demand side, bidders are not allowed to reduce the prices in their submitted bid schedules but are allowed to increase the prices in their bid schedules. As a result, the demand schedule is weakly increasing by construction. Using the notation of derived supply and demand in round *t* from the previous section, these arguments establish $\frac{\partial S_i^t\left(p_i^t\right)}{\partial p_j^t} \leq 0$ and $\frac{\partial D_i^t\left(p_i^t\right)}{\partial p_j^t} \geq 0$. Combining these inequalities, the excess demand in market *i* is weakly increasing in response to changes in the price of other markets, establishing the gross substitute property: $\frac{\partial\left[D_i^t\left(p_i^t\right)-S_i^t\left(p_i^t\right)\right]}{\partial p_j^t} \geq 0$.

Having established the gross substitutes property, we now relate price adjustments to a tatonnemont-like process driven by excess demand. As in the model of the Walrasian auctioneer, all provisional allocations and prices are announced based on reported demand with no actual trading taking place, a key feature of tatonnemont. Through revisions to the submitted bid schedules, prices mechanically increase in response to excess demand. Since excess demand induces revealed demand at the margin, it causes reported demand to shift upward against an inelastic supply curve. This price increase is directly attributable to excess demand in the market, and only in that market, for that license.

Section 6 presents a complete discussion of the time-series properties of the bidding data. Having established convergence of allocations, prices, and revenues, we empirically estimate the adjustment process for prices in response to excess demand. We test for cross-market effects in how prices for a specific license respond to excess demand for other types of gambling licenses. We also demonstrate the estimated adjustment matrix satisfies stability conditions for convergence, providing empirical evidence to support the theoretical arguments that the system attains a general equilibrium and an efficient allocation of licenses across markets.

[12] This classical economic result goes back to Arrow and Hurwicz (1958) and Hahn (1958) and is commonly presented in core textbooks such as Arrow and Hahn (1971), Mas-Colell, Winston, and Green (1995), and McKenzie (2002). Analyzing markets with indivisibility is considered by Kelso and Crawford (1982), who identify the gross substitutes property as a sufficient condition for the existence of Walrasian equilibria with indivisible goods. Gul and Stacchetti (1999) study the efficiency of Walrasian equilibrium in economies with indivisibilities satisfying gross substitutes.

KEY RESULT 5.3: Licenses satisfy the gross substitutes property, suggesting according to classical general equilibirum theory that tatonnemont process can yield an efficient allocation with equilibrium prices.

Section 6: Implementation, Revenue, and System Convergence

We now discuss the system's performance in terms of the total revenue generated from distributing licenses, the prices realized for licenses sold in different market segments, and the reported dynamics of demand. The government's original plan was to allocate all 27,500 entitlements in the auction, to take place in early 2010. The government later decided, after the auction rules had been largely designed, to give existing clubs an opportunity to buy a capped number of entitlements at a set price. Eligible clubs could buy an entitlement for each gaming machine currently operating at their venue, up to a cap of 40 entitlements per venue. The offer price was based on a percentage of the individual venue's historic gaming revenue, and thus differed across clubs. Most clubs (236 out of 247 eligible) bought at least one entitlement in this phase, and in aggregate they purchased 8,712 of the 13,750 entitlements available to clubs (63 percent). Following this pre-auction sale, which took place in October and November of 2009, 5,038 club entitlements remained for sale in the auction along with the original 13,750 entitlements available for hotels.

The auction took place in two phases. First was an initial round of bid submissions without price or bid revelations that was open for two weeks. During this period an individual bidder could examine or change own bids as practice with the bidding and information interface. Prices were computed and revealed at the beginning of a one-day, one-site auction that lasted for about 10 hours and an additional 62 rounds. Bidders were required to submit a bid in the initial round in order to be eligible to participate in the later one-day auction that finalized the entitlement allocation. The initial two-week time period for bid submission was similar in all respects to the bidding features of later rounds, but was open for many days to ensure that interested bidders had an opportunity to become familiar with the auction interface, consult with advisors when constructing their initial round bid, and communicate with "coaches" who provided technical assistance with bid preparation and submission. All bids in round one were communicated to the auctioneer through secure internet connections and prices and allocations were announced for the first time at the opening of the auction day.

After the two-week initial round was completed, bidders were required to bid on-site at a secured convention center in Melbourne for phase two. Upon check-in they were assigned to a bidding station, which contained a visually isolated computer workstation and seating for a bidding "team" of up to two individuals as illustrated in Figure 8. A total of 363 bidding teams participated in the on-site auction. Bidder cell phones were collected and they had no access to public phones. Bidders could not walk through the bidding area unmonitored. They could talk to the bidder on their own team but not bidders on other teams. Coaches were assigned and available for assistance or to interface with auctioneers should problems arise.

Bidders were registered according to the markets in which they wanted to place bids. As explained earlier, bidders submitted a schedule that indicated the marginal quantities that they were willing to pay at various market prices of their choosing. At the close of each round provisional prices were calculated and publicly announced for each market, and bidders learned their provisional allocation and prices per entitlement won. Market volumes were not announced publicly. Between rounds bidders could revise upwards any bids subject to a minimum price increment. They could also cancel bids that were not provisionally winning. Rounds continued as long as the closing was not triggered by insufficient new bids or winning bids as explained previously.

Figure 8: Bidding Stations in Melbourne Convention Center

The auction performed smoothly and ended as planned, and there were no technical difficulties. Price discovery was slower than seen in the testbed experiments, but this is due to the auction's large size. Unlike in an experiment with induced values, of course the underlying entitlement values are not observable. Nevertheless, based on an analysis of the bidding behavior we conclude that prices and allocation quantities appear to reflect the conditions of an efficient allocation. The auction permitted entry by new venues as anticipated, since bidders other than existing venues were successful in acquiring entitlements.

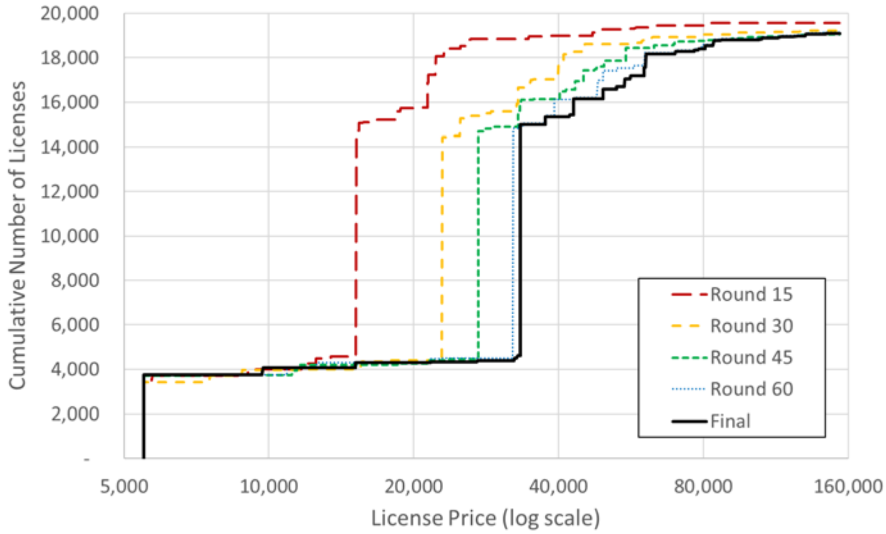
6.1 Price Dynamics and Revenue Convergence

In its final allocation, the system allocated 18,788 licenses to 428 establishments, generating a total revenue of AU$615M, or about US$555M based on prevailing exchange rates at the time. Table 3 presents summary statistics on the number of bidders in each market, the number of areas assigned their maximal allocations, and the components of the system's total revenue. Representing more bidders submitting higher total valuations, Hotel establishment licenses were capped at their maximal total allocation of 13,750 units. The remaining licenses were allocated to clubs, which collectively generated 12% of the system's revenue. Only sixteen areas' allocations (out of 88 total) reached the constrained maximum allowance, but due to the price premium for these licenses, these areas represented nearly 41% of the revenue for the system. On average, these "Constrained" areas also featured a relatively large number of bidders.

Table 3: Active Bidders, Total Value, Allocations, and Revenue Goes Here

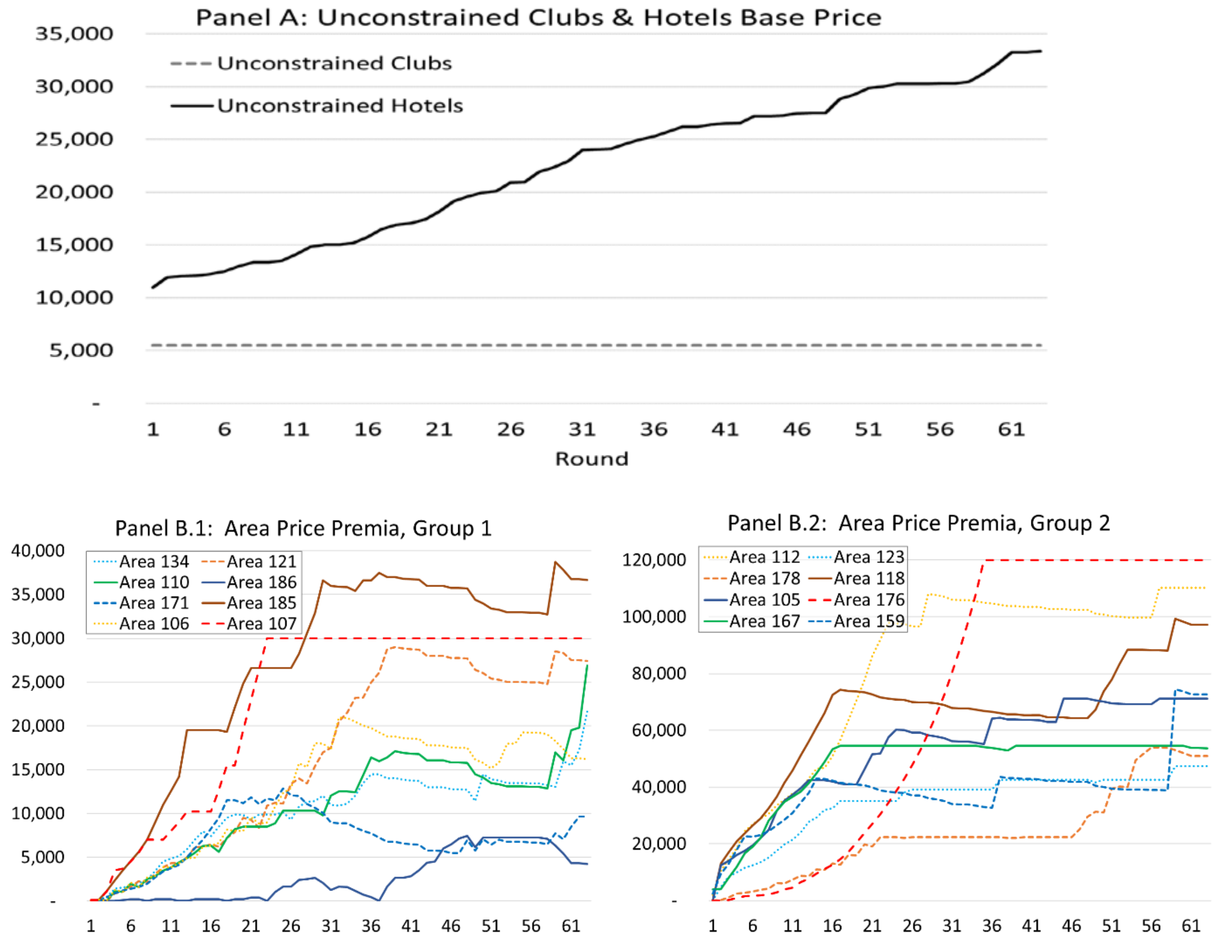
Clubs	**Number of Bidders**	**Number of Areas**	**Total Bid Quantity**	**Total Bid Value**	**Awarded Quantity**	**Average Price**	**Total Revenue**
Unconstrained	103	72	3,757	58,688,745	3,693	5,500	20,311,500
Constrained	64	16	1,617	105,545,885	1,345	36,558	49,171,047
Clubs Total	167	88	5,374	164,234,630	5,038	13,792	69,482,547
Hotels							
Unconstrained	184	72	12,751	498,270,158	10,356	33,350	345,372,600
Constrained	77	16	4,479	280,848,522	3,394	59,019	200,311,424
Hotels Total	261	88	17,230	779,118,680	13,750	39,686	545,684,024
Overall Total	428	176	22,604	943,353,310	18,788	32,743	615,166,571

Figure 9: Cumulative Interim Quantities Allocated by Price



To demonstrate the heterogeneity in prices for different types of licenses as the auction progressed, Figure 9 presents the cumulative distribution of prices for each license in the provisional allocations for rounds 15, 30, 45, 60, and the final allocation. Note the dramatic differences in quantities of licenses at nearby prices held by bidders, and the atomic nature of the distribution, which results from the uniform pricing by the system for all bidders in the same market. The two largest atoms, the largest differences, in this distribution correspond to licenses sold to club and hotel bidders in areas with an allocation strictly less than the constrained maximum. Most of the heterogeneity in prices, therefore, is attributable to prices in areas where the allocation is constrained. As the auction progressed, prices increase and the distributions of prices for later rounds dominate those of earlier rounds. In the final rounds, the distribution of prices in round 60 is very similar to the final distribution at the end of round 63 due to the system's equilibration as submitted bid schedules stabilize in later rounds. Though prices for clubs bidding in unconstrained areas remained constant throughout the system's operation, the price for hotels and bidders in constrained areas shifts dramatically as the system progressed.

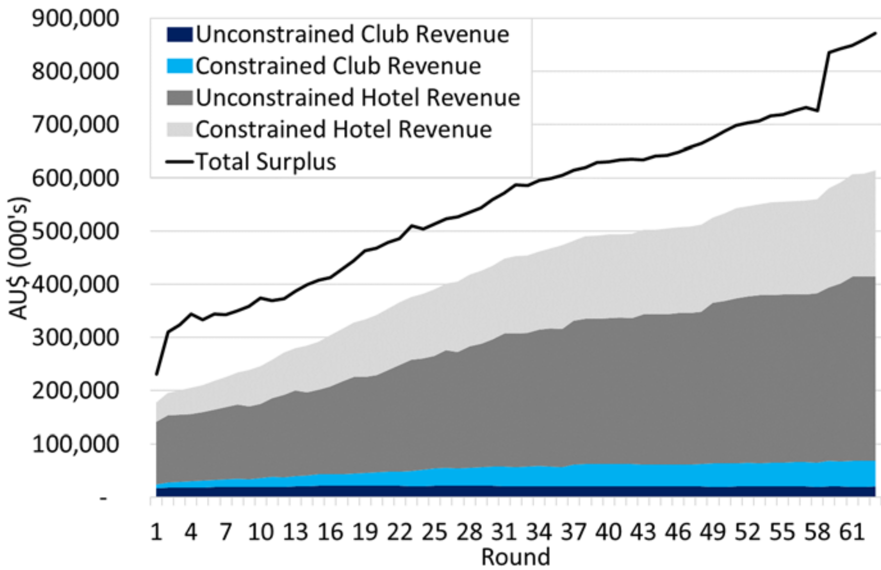
Figure 10: Time Series of Prices in Unconstrained and Constrained Markets



For additional perspective on the dynamics of interim prices and allocations, consider the time series of provisional prices across rounds in individual markets presented in Figure 10. As demonstrated by Figure 9, the majority of licenses were sold to clubs and hotels in areas that were not constrained by maximum allocations, so these series are presented separately in Figure 9, Panel A. Panels B1 and B2 of Figure 10 plot the time series of interim price premia for clubs and hotels in areas where the allocation was subject to maximum constraints. Each of these plots demonstrates an upward trend in prices in early rounds that then levels off in later rounds. Two exceptions to this pattern are noteworthy. The first represents interim prices for club licenses in unconstrained areas, which stayed constant throughout the system's operation. The second corresponds to the area price premia in areas 03 and 81, which featured a large jump in prices during a relatively late round 59. As figure 10 makes clear, however, the last round only changed area price premia for the markets in areas 01 and 05.

Figure 11 presents the time series of total revenue realized by the system's provisional allocations, broken down by market segments (unconstrained clubs, unconstrained hotels, constrained clubs, and constrained hotels). The solid black line plots the total reported surplus, based on the total stated willingness to pay from submitted bid schedules.

Figure 11: Time Series of Total Revenue from Interim Allocations



KEY RESULT 6.0: Presentation of summary statistics and total revenues. Prices and revenue converge as the auction progresses towards its final round.

Section 7: Estimating Price Dynamics and Testing Stability

As suggested earlier, the multi-round bidding format gives rise to a price adjustment process analogous to tatonnement. Many models of disequilibrium price dynamics, including tatonnemont, link price adjustments to the magnitude of excess demand. By exploring the dynamic relationship between price changes and the revealed demand at the margin, we can learn about the nature of equilibration in these markets. In particular, we can verify that the dynamic system of price adjustments satisfies classical conditions for stability that lead tatonnemont processes to converge on the general equilibrium leading to a deeper understanding of the multi-market equilibration process. This analysis builds upon the gross substitutes property for licenses from section 5.3 that provided theoretical conditions for general equilibrium stability of the auction across markets. Here, we demonstrate the observed empirical process of price adjustment satisfies conditions for stability and general equilibrium convergence to prices supporting an efficient allocation across markets.

Classical analysis of general equilibrium systems characterizes how prices equilibrate in response to excess demand[13] by differentiating excess demand functions $z_t(p)$ with respect to prices:

$$\frac{\delta}{\delta t} z_t(p) = \nabla_p z_t(p) \frac{\delta}{\delta t} p_t + \varepsilon_t = B^{-1} \Delta p_t + \varepsilon_t \tag{4}$$

Because $B^{-1} = \nabla_p z_t(p)$ is the gradient of current excess demand with respect to prices, the matrix B is sometimes referred to as the Inverse Jacobian of the excess demand function. Solving for Δp_t in equation (4) and letting $u_t = B\varepsilon_t$ represents the price adjustment process as a linear function of excess demand that resembles a regression equation.

$$\Delta p_t = B z_t(p) + u_t \tag{5}$$

McFadden (1960) provides sufficient conditions for the matrix B to characterize a stable system. These conditions impose two restrictions on the coefficients that predict price movements in response to excess demand. First, the diagonal entries in B must be positive, so that $\beta_{mm} > 0$ for all m. Second, the determinant of the (m, m) principal minor for the matrix B must be weakly negative. We can thus evaluate whether the system's response to excess demand is stable by testing these two conditions.

Here, we pose two questions. First, as an empirical characterization of equilibration dynamics, do observed prices adjust according to measures of excess demand? Specifically, if we fit data to equation (5), do we find a significant relationship between price changes and excess demand? Second, does this Empirical Inverse Jacobian satisfy the conditions for stability that would lead to price convergence to equilibrium according to classical analysis of multi market economic systems? These conditions are well understood from a purely theoretical perspective. The rich data available in the Victoria Gaming Auction allows us to investigate these basic and important theoretical properties in an empirical setting.

[13] For brief outlines, see Negishi (1989) for a discussion of tatonnement and its theoretical background. Reviews of general equilibrium theory can also be found in McKenzie (1989) and Fisher (1989). More extensive discussions of the gross substitute case are found in Arrow and Hahn (1971), Negishi (1972, 1959), Mukerji (1995, 2002), and Murkerji and Guha (2011). Experimental work on stability of general equilibrium can be found in, Hirota, Hsu, Gillen, Plott, and Rogers (2018), which demonstrates that double auction markets that do not satisfy general equilibrium stability conditions follow excess demands and diverge from an interior competitive equilibrium.

7.1 Estimating Price Response to Excess Demand

We begin by considering the first classical condition for stability of the system presented in equation (5). This condition requires the price for a given type of license respond positively to excess demand for that license. Following the analysis from Section 4, the constraints on the system effectively generated eighteen (18) different market segments or types of licenses: Unconstrained Clubs, Unconstrained Hotels, and sixteen (16) Constrained Areas with Area Price Premia. Let N denote the set of derived markets,[14] and let $z_{nt}(p_{t-1})$ denote the excess demand in market n and round t. Consider the price adjustment process for market m, $\Delta p_{mt} = p_{mt} - p_{mt-1}$:

$$\Delta p_{mt} = \beta_{m1} z_{1t}\left(p_{t-1}\right) + \cdots + \beta_{mm} z_{mt}\left(p_{t-1}\right) + \cdots + \beta_{mN} z_{Nt}\left(p_{t-1}\right) + u_{mt} \tag{6}$$

The regression equation (6) relates the expected price changes for market m to the level of disequilibrium in each of the individual markets including the excess demand in m itself, providing a predictive model for price changes driven by excess demand.

Estimating the model requires first specifying a measure of excess demand for each market in each round. "Closing" prices, or the prices associated with any given allocation, are determined to clear markets. Therefore, zero excess demand exists based on stated bid functions at closing prices and provisional allocations. The Revealed Excess Demand at the Margin introduced in section 4.2, however, provides a natural proxy of excess demand at the opening price for a round. The measure takes advantage of reported demand at two different price points where bidders' incentives are binding, at least in determining provisional allocations. As Revealed Demand is reported in terms of licenses for each specific market, these measures need to be translated into a common numeraire for evaluation across markets. Consequently, we measure excess demand by scaling revealed demand at the margin by the opening price of the round:

$$z_{mt}\left(p_{t-1}\right) = p_{m,t-1} \sum_{a,v \in m} \Delta D_{av}^{t}\left(p_{t-1}\right)$$

Table 4 reports the estimated values for the coefficients β_{mm} along with their standard errors, with estimates for all parameters appearing in Appendix B.[15] The estimated coefficients

[14] Here, and henceforth, we will use "market" to refer to a derived market as defined in Section 4. That section identified Unconstrained Clubs as representing one market that determines the Unconstrained Club Base Price, Unconstrained Hotels as another that determines Hotel Premium, and fifteen Constrained Area Markets that determine the Area Premia paid by all bidders within each of these areas. Since no variation in price or excess demand exists for the Unconstrained Clubs, we exclude that market from the analysis.

[15] As discussed in the next section, this regression equation represents a system of Seemingly Unrelated Regressions. We estimate these regressions on an equation-by-equation basis in this section to focus on each individual market's price response to excess demand within that market. In the next section, when we consider cross-market restrictions necessary for stability, we exploit the SUR structure to establish joint asymptotic

suggest that prices typically respond positively to excess demand, given 12 of the 17 estimated β_{mm} coefficients are positive and none of the negative estimates are significantly negative. These results suggest an affirmative answer to our first question investigating whether prices respond to excess demand. Further, the direction of price response is consistent with theory that suggests positive excess demand leads to rising prices while prices drop in response to negative excess demand.

Table 4: Price Adjustment in Response to Excess Demand

	UC Hotel	**Area 105**	**Area 106**	**Area 107**	**Area 110**	**Area 112**
$\hat{\beta}_{mm}$	1.87E-05	1.57E-02	-1.81E-04	1.19E-04	-2.72E-04	2.09E-04
Std Error	4.07E-06	5.21E-03	1.30E-04	2.39E-04	1.11E-03	6.78E-04
t-Stat	4.60**	3.01	-1.39	0.50	-0.24	0.31
	Area 118	**Area 121**	**Area 123**	**Area 134**	**Area 159**	**Area 167**
$\hat{\beta}_{mm}$	6.83E-04	2.55E-04	-3.92E-04	2.69E-05	-1.53E-02	2.20E-03
Std Error	6.16E-04	2.55E-04	2.45E-04	2.16E-05	2.80E-02	9.36E-04
t-Stat	1.11	1.00	-1.60	1.25	-0.55	2.35
	Area 171	**Area 176**	**Area 178**	**Area 185**	**Area 186**	
$\hat{\beta}_{mm}$	9.23E-05	-2.65E-04	1.45E-04	2.76E-04	2.05E-05	
Std Error	1.71E-04	7.66E-04	4.79E-04	2.14E-04	1.72E-04	
t-Stat	0.54	-0.35	0.30	1.29	0.12	

A practical challenge arises in analyzing the regression specification in (6) due to the need to fit 17 coefficients with only 62 rounds of data for each of the 17 market segments (recalling the unconstrained club market is excluded due to lack of variation in price and excess demand). Given the number of free parameters in regression equation (6), one might be concerned that the model is overparameterized as evidenced by the relatively large standard errors and limited significance of estimates in Table 4. To reduce the dimensionality of the problem, we evaluate price changes with respect to the excess demand for licenses in a single market and the total excess demand for licenses in all other markets. That is, define the composite measure of excess demand for other markets as $z_{(-j)t}\left(p_{t-1}\right)=\sum_{n\neq j} z_{nt}\left(p_{t-1}\right)$ and consider the simplified regression:

$$\Delta p_{mt}=\beta_{m1}z_{mt}\left(p_{t-1}\right)+\beta_{m2}z_{(-m)t}\left(p_{t-1}\right)+\varepsilon_{mt} \tag{7}$$

normality of all estimates and justify a parametric bootstrap as a device for calculating standard errors for inference.

This specification concentrates the influence of excess demand across other markets, reducing the dimensionality of the regression to enable more precise estimates.

Table 5 represents the results for estimating regression specification (7) using a maximum likelihood mixed effects model for heterogeneous coefficients across markets (Panel A) and OLS fixed effects model (Panel B). For ease of interpreting the coefficients, all independent and dependent variables are standardized to mean zero and unit variance by market. Overall, the results demonstrate that positive excess demand is associated with an expected increase in prices in both the mixed-effects and fixed-effects specifications. Prices respond positively to market-specific excess demand in 11 of the 17 markets featuring positive estimates for β_{m1} with t-Statistics passing the traditional threshold for 5% significance in eight of these markets. The mixed effects model consolidates these results, further verifying the expected positive sign of β_1 consistent with theoretical restrictions of stability. This robustness result further affirms the results demonstrated in Table 4: the empirical evidence shows that prices change in response to excess demand as posited by theoretical analysis.

Table 5: Excess Demand and Price Dynamics

Panel A: Random Effects Pooled Results Across Markets

Coefficient	Estimate	Std Error	t-Stat	Random Effect Std Deviation
$\hat{\beta}_1$	0.205	0.066	3.12	0.24
$\hat{\beta}_2$	0.178	0.046	3.88	0.14

Panel B: Fixed Effects Individual Market Results

Market	$\hat{\beta}_{j1}$	t-Stat	$\hat{\beta}_{j2}$	t-Stat	Market	$\hat{\beta}_{j1}$	t-Stat	$\hat{\beta}_{j2}$	t-Stat
UC Hotel	0.335	2.32	0.025	0.17	Area 134	0.595	5.30	-0.081	-0.72
Area 105	0.516	4.17	0.131	1.06	Area 159	-0.073	-0.50	0.322	2.18
Area 106	-0.054	-0.42	0.200	1.55	Area 167	-0.023	-0.19	0.507	4.17
Area 107	0.261	1.90	0.160	1.17	Area 171	-0.021	-0.13	0.297	1.89
Area 110	0.289	2.31	-0.004	-0.03	Area 176	-0.301	-1.98	0.261	1.71
Area 112	0.587	5.23	0.096	0.86	Area 178	-0.123	-0.87	0.014	0.10
Area 118	0.512	5.82	0.440	5.00	Area 185	0.288	2.55	0.372	3.29
Area 121	0.000	0.00	0.151	0.99	Area 186	0.077	0.61	-0.171	-1.35
Area 123	0.532	5.72	0.371	3.99					

Interestingly, Table 5 also demonstrates significant sensitivity for prices to respond to excess demand in other markets. Four of the individual-market regressions demonstrate a significant price response to aggregate excess demand in other markets. Further, the estimated value of β_2 in the random effects model is positive and highly significant. These estimates provide evidence of general equilibrium dynamics in the system that cause prices to shift in response to excess demand in other markets and raise the question of whether these effects might lead to instability in the price adjustment process.

KEY RESULT 7.1: In the Victoria Gaming Auction, whether for licenses in unconstrained areas for hotels or for constrained area price premia, prices for each license type respond positively to excess demand for those licenses. That is, high excess demand for a license type causes its price to rise more quickly than low excess demand. This result validates the first classical condition of stability for dynamic price systems

7.2 Testing Stability of Price Adjustment Process

Given our measures of excess demand and observed interim prices, we can treat tatonnemont as an empirical process rather than a purely theoretical construct. That is, treating the dynamic price adjustment process specified in equation (5) as a regression equation allows us to estimate the Empirical Inverse Jacobian for the price adjustment process. By estimating the rates of adjustment for prices in response to observed excess demand, we can estimate and test the hypothesis that observed price adjustments are consistent with classical restrictions of stability under which prices converge to equilibrium.

The regression specifications (6) and (7) also provide a device for evaluating the stability of the observed price dynamics. A variety of classical models for disequilibrium price dynamics characterize the system of price dynamics in response to prevailing excess demand in multiple markets. These models consider the full system of price changes:

$$\begin{bmatrix} \Delta p_{1t} \\ \vdots \\ \Delta p_{Nt} \end{bmatrix} = \Delta p_t = B z_t\left(p_{t-1}\right) + \varepsilon_t = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1N} \\ \vdots & \ddots & \vdots \\ \beta_{N1} & \cdots & \beta_{NN} \end{bmatrix} \begin{bmatrix} z_{1t} \\ \vdots \\ z_{Nt} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \vdots \\ \varepsilon_{Nt} \end{bmatrix} \tag{8}$$

Recalling McFadden (1960)'s two conditions for the matrix B to characterize a stable system, we test if (1) the diagonal entries in B are positive, so that $\beta_{mm} > 0$ and (2) the determinant of the (m, m) principal minor for the matrix B is weakly negative.

We have already evaluated condition (1) in the previous subsection, as summarized by Key Result 7.1, with the results in Tables 4 and 5 provide supportive evidence that prices respond positively to excess demand within a market. Indeed, for the mixed effects model suggests fewer than 20% of samples will yield negative estimates for β_{m1}, which corresponds to the β_{mm} coefficient in model (6). Considering the regression estimates presented in Table 7.1, we note positive coefficient estimates suggest that the prices increase with excess demand in twelve of the seventeen markets. While there are five markets with negative coefficient estimates, suggesting that prices actually fall in the presence of significant excess demand, none of these are statistically significant.

The second condition for determining stability is somewhat more challenging to test and, to our knowledge, has never been performed before a multiple market economic system. From an econometric perspective the test is facilitated by the Seemingly Unrelated Regression (SUR) representation implied by (8). Given estimates of the coefficients in the matrix B, computing the determinant of the principal minors for each of its diagonal elements is straightforward. Since the determinant is a continuous function, we can apply a delta-method approximation to establish asymptotic normality and use a parametric bootstrap to construct confidence intervals. The details of this inference technique, including the parametric bootstrap, are reported in Appendix C.

Table 6: Estimates and Bootstrap Confidence Intervals for Determinant Test of Stability

Panel A: Bootstrap Confidence Interval for
Maximum Principal Minor Determinant across Markets

Lower 95%	Expectation	Upper 95%
-1.14E-42	3.05E-43	1.75E-42

Panel B: Bootstrap Confidence Interval for Principal Minor Determinants by Market

Market	Lower	Estimate	Upper	Market	Lower	Estimate	Upper
Uncon Hotel	-2.17E-42	1.26E-44	2.19E-42	Area 134	-2.32E-44	-7.17E-47	2.31E-44
Area 105	-1.77E-44	1.55E-48	1.77E-44	Area 159	-1.43E-45	9.70E-48	1.45E-45
Area 106	-4.59E-44	-1.14E-46	4.56E-44	Area 167	-1.07E-45	-1.33E-48	1.07E-45
Area 107	-3.73E-45	4.38E-48	3.73E-45	Area 171	-6.80E-44	-4.09E-47	6.80E-44
Area 110	-2.31E-44	-1.37E-47	2.31E-44	Area 176	-4.88E-45	-1.92E-48	4.88E-45
Area 112	-1.25E-44	2.31E-47	1.26E-44	Area 178	-1.60E-44	-4.43E-47	1.59E-44
Area 118	-1.70E-44	2.60E-47	1.71E-44	Area 185	-3.02E-44	4.26E-47	3.03E-44
Area 121	-1.46E-44	8.51E-48	1.46E-44	Area 186	-1.10E-43	3.29E-46	1.10E-43
Area 123	-6.05E-44	-6.56E-47	6.03E-44				

Table 6 presents the estimates of the relevant determinants for each market, along with their 95% confidence intervals. Panel A characterizes confidence intervals for the maximum of the determinants across all principal minors of the B matrix from Regression (8). Though the bootstrapped mean of this statistic is positive, it is quite close to zero and an order of magnitude smaller than the bootstrapped standard error of the estimate so that the 95% confidence interval clearly includes zero. An asymptotic approximation assigns a p-value of only 0.34 for the null hypothesis that the maximum determinant is weakly negative, which does not support rejecting the second condition of stability at any reasonable level.

Panel B of Table 6 presents the estimated principal minor determinants of the Empirical Inverse Jacobian for each market along with their 95% confidence intervals. Though many markets' point estimates for these determinants are positive, the 95% confidence interval always includes zero in every case and their magnitudes are extremely small. One challenge associated with the test relates to the relatively large number of parameters involved relative to the number of rounds for which we have bidding data, and the large standard errors suggest the test features limited power in this sample.

Overall, our analysis of the Empirical Inverse Jacobian of excess demand demonstrates that the price dynamics observed in the Victoria Gaming Auction are consistent with a stable equilibration process. First, prices in a market shift positively in response to excess demand, increasing when excess demand in the market is positive and decreasing when excess demand is negative. Second, excess demand in other markets does not generate unstable general equilibrium effects in the price adjustment process. Showing the Empirical Inverse Jacobian statisfies these stability conditions provides further evidence that the price adjustment process leads to a stable equilibrium in prices. Beyond informing the scientific interest in empirically investigating conditions for tatonnemont convergence, this stable equilibrium further supports the efficiency of the final allocation achieved by the auction mechanism.

KEY RESULT 7.2: The Empirical Inverse Jacobian suggests observed price dynamics in the Victoria Gaming Auction are consistent with a stable general equilibrium system that will converge.

Section 8: Conclusion and After Market Evaluation

This paper reviews the design and implementation of an auction mechanism to solve a complex allocation problem that involved 176 interdependent markets and prices, 18.788 entitlements and 363 bidders. The allocation problem was dictated by social policies that led to constraints on the distribution of gambling activities in a highly regulated industry. Addressing these government concerns presented a challenging policy design problem. The Victoria Gaming Auction approached this problem by starting with theoretical properties of an efficient allocation, identified as the solution to a constrained surplus maximization problem. The performance of the mechanism, including practical elements of its function, rules, and technical issues were determined and refined through extensive testbedding in an experimental setting. The successful transition from the lab to the field is supported by theoretical principles, and verified by the empirical properties from the time-series of observed price dynamics and excess demand. The mechanism itself is based on competitive economic theory with practical features suggested by some of the prominent features of the classical tatonnement theory of price adjustment and refined through an experimental testbedding process.

In the end, we demonstrate that the mechanism achieved the basic design and assessment goals. First, it did what is was designed to do. The resulting allocation satisfied basic properties of efficiency subject to the fact that the complex legal, political and social goals were met. Secondly, the data demonstrate that the design success can be attributed to the underlying principles from which the design was constructed. The results were not a consequence of luck or some arbitrary random events.

A competitive theory of general equilibrium related behavior underlies the principles determining allocations within the mechanism. If individuals are willing to buy more entitlements at a stated price, they can attempt to obtain additional licenses by simply increasing their bids. The decisions result in measurements naturally interpreted as market demand functions. The response of the auction to such revealed demand functions and resulting excess demands was to adjust allocations and thus prices to reflect an efficient allocation and equilibrium supporting prices given participants' values revelation. In the last round of the auction's operation, only five bidders made small adjustments to their bid schedules, suggesting that bidders were satisfied with their allocations at the prevailing prices. In effect, participants' unwillingness to revise their bid schedules suggests an ex post efficiency to the final allocation of licenses.

The bidding process also provided new insights into the dynamics by which demand and excess demand are revealed through the gaming entitlements auction mechanism. Since bidders' incentives bind only at the interim prices and allocations announced between rounds, it is at these prices and allocations where bidders' demand is truthfully revealed. This principle of "demand revelation at the margin" allows us to measure excess demand and demonstrate that excess demand diminishes as bidding rounds progress and prices increase. The property is closely related to a general principle treated as a theoretical axiom since Walras that if excess demand of a commodity is negative then other things being equal, its price will fall.[16]

Finally, we observe how relative prices evolve across bidding rounds in response to these revealed excess demands. This provides a unique opportunity to evaluate classical properties of multi-market, equilibrating dynamics. We define the Empirical Inverse Jacobian as the empirical counterpart to the inverse Jacobian of excess demand governing price dynamics under tatonnemont price adjustments. We estimate this Empirical Inverse Jacobian using the Victoria Gaming Auction interim prices, bids, and allocations to compute observed price changes and imputed excess demand. We derive tests to show the Empirical Inverse Jacobian satisfies classical conditions for stability, with results supporting the hypothesis that prices converge toward their equilibrium values as revealed by the model. These results demonstrate the importance and power of classical general equilibrium theory in addressing real-world market design problems.

[16] See the discussion in Mukerji (2002),p.74, or in McKenzie (2002) p.54. Walras (1954, p.170) notes the property as fundamental: "If the demand for any one commodity is greater than the offer, the price of that commodity in terms of the numeraire will rise; if the offer is greater than the demand, the price will fall."

In sum, the Victoria Gaming Auction delivered a stable and efficient allocation for gaming licenses across a large number and a variety of markets and to a variety of establishments. This allocation satisfied policy constraints while generating revenues of AU$614 million for the Victoria Government in a ten-hour period plus an additional AU$366 million from the pre-auction offer to existing bidders – a total of AU$980 million. Its success demonstrates the effectiveness of combining economic theory with experimental testbedding in applied mechanism design. It also shows the usefulness of using laboratory experimental techniques for revealing the content and meaning of basic economic principles in the context of a multidisciplinary and politically and legally sensitive policy. Finally, evaluating the mechanism's performance and analyzing the time series of prices and demand provides new insights into the market and excess demand forces, at the heart of general equilibrium theory, driving price equilibration in multiple market settings.

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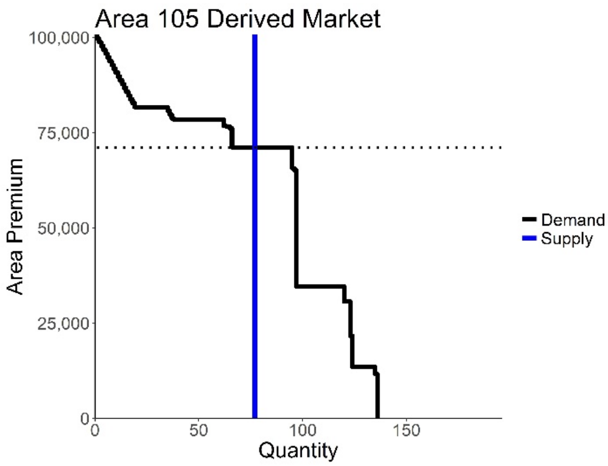
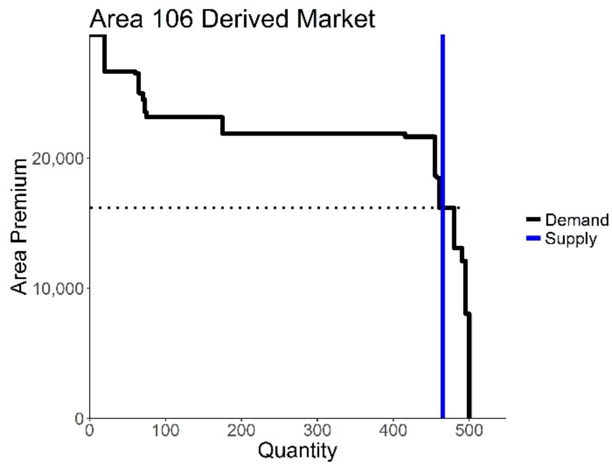
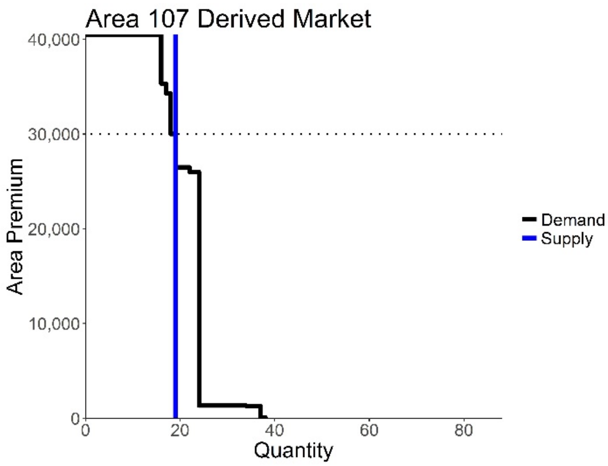
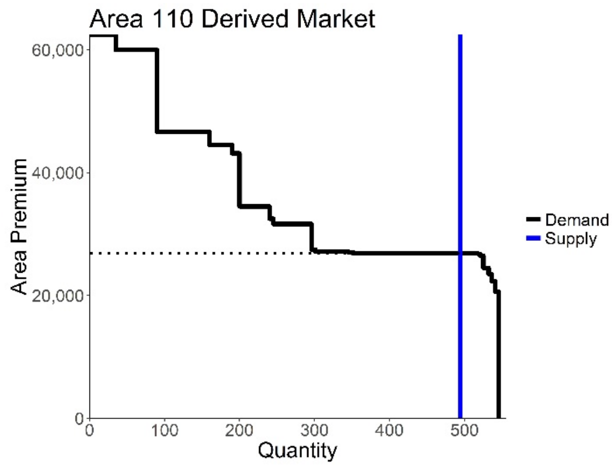
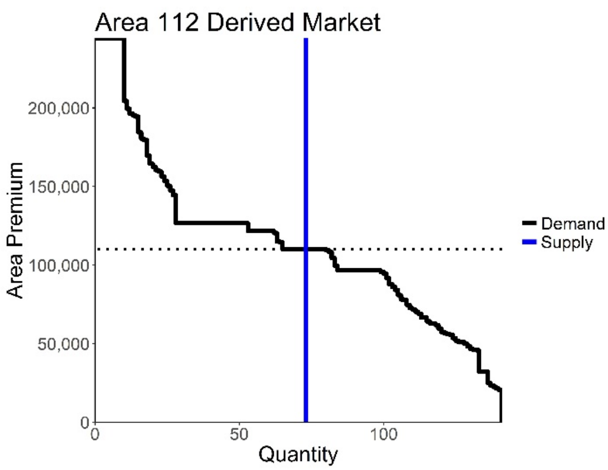
Appendix A: Detailed Demand and Supply Figures for Area Price Premia

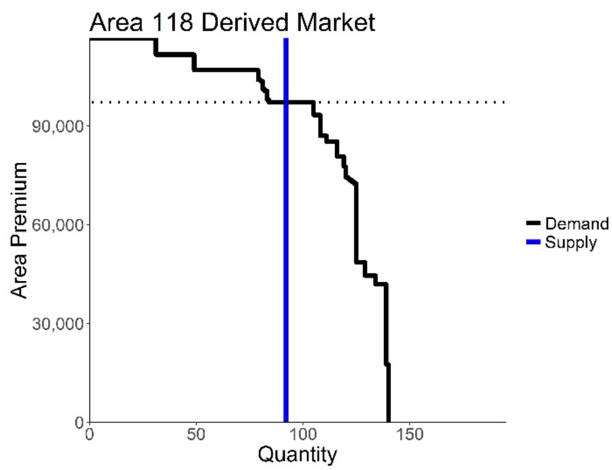




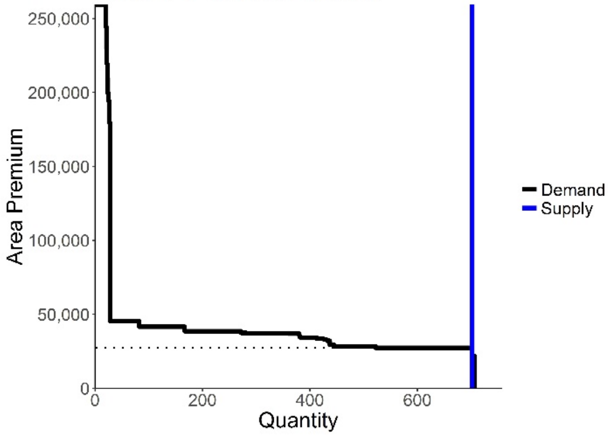




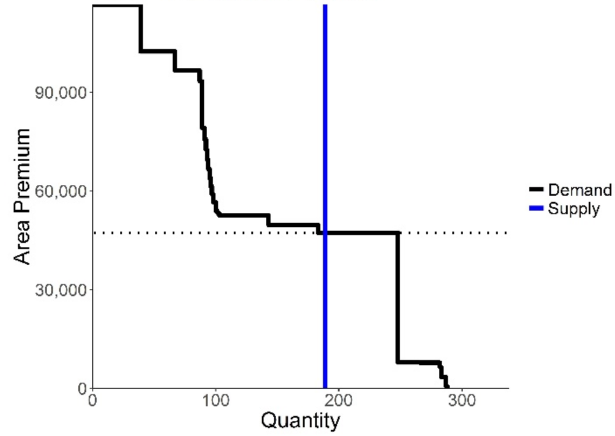




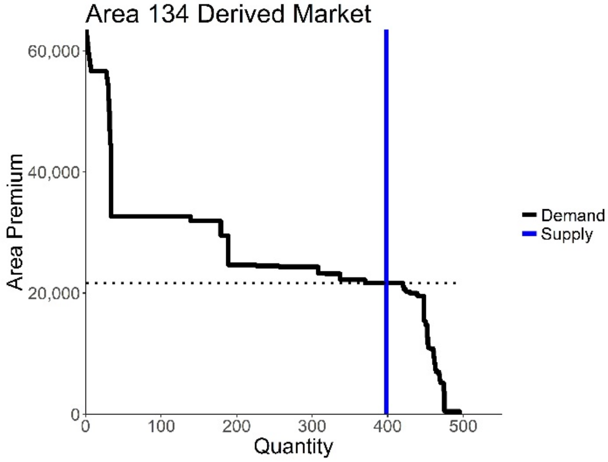
Area 121 Derived Market



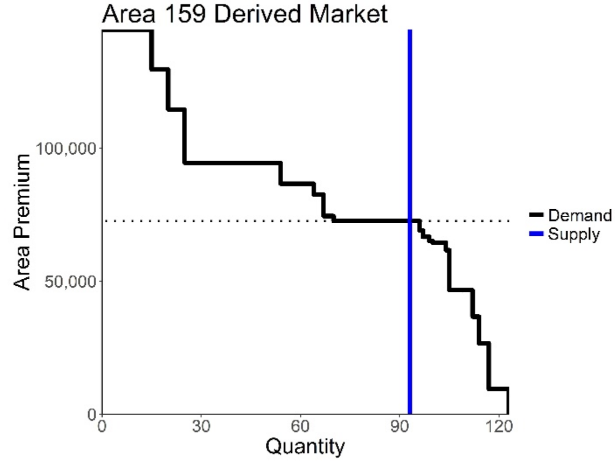
Area 123 Derived Market



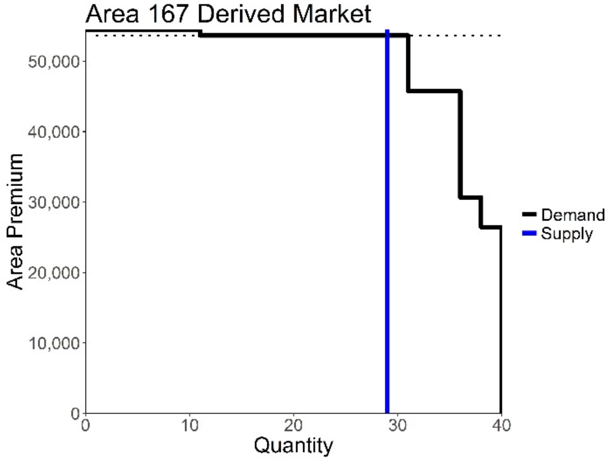
Area 134 Derived Market



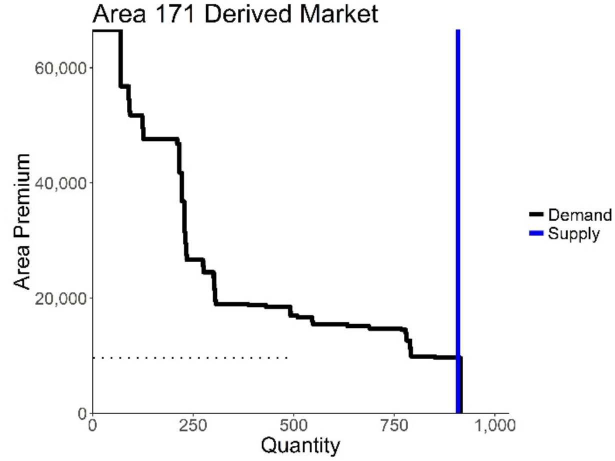
Area 159 Derived Market

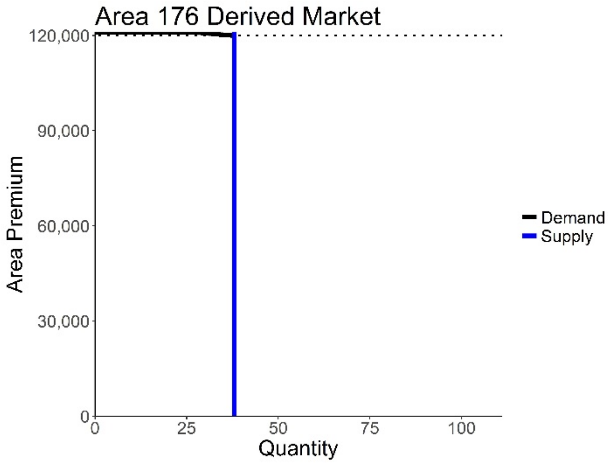


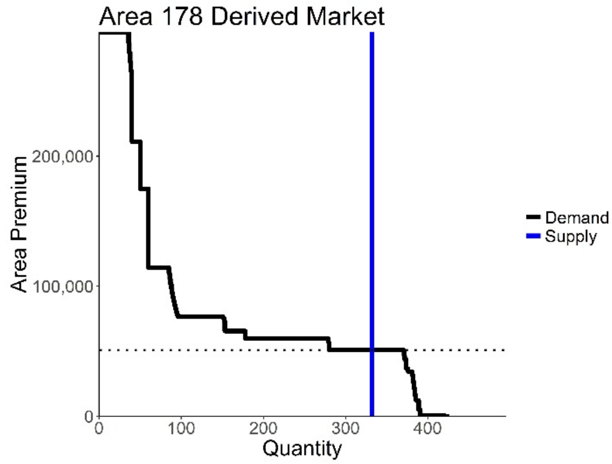
Area 167 Derived Market

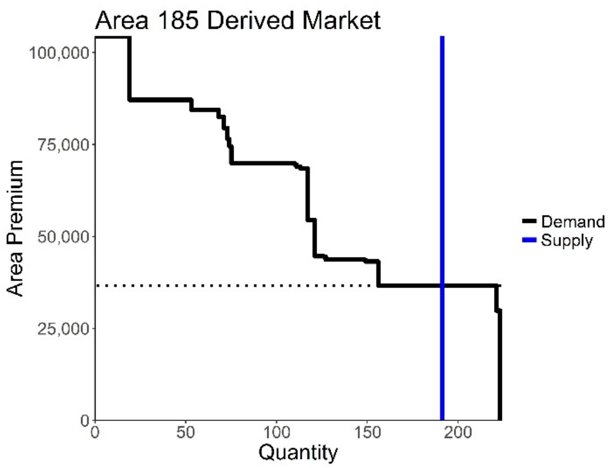


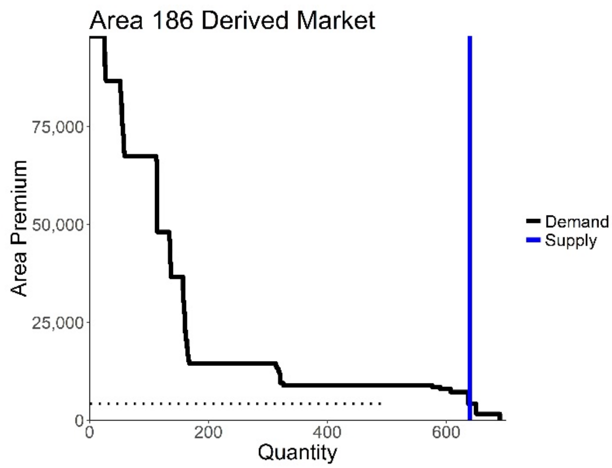
Area 171 Derived Market




Area 176 Derived Market
120,000
90,000
60,000
30,000
0
Area Premium
0
25
50
75
100
Quantity
Demand
Supply


Area 178 Derived Market
200,000
100,000
0
Area Premium
0
100
200
300
400
Quantity
Demand
Supply


Area 185 Derived Market
100,000
75,000
50,000
25,000
0
Area Premium
0
50
100
150
200
Quantity
Demand
Supply


Area 186 Derived Market
75,000
50,000
25,000
0
Area Premium
0
200
400
600
Quantity
Demand
Supply

Appendix B: Full Regression Results for Regression Model 7.3

		Revealed Excess Demand for Licenses in Market																
Market Price Response		*UC Hotel*	*Area 105*	*Area 106*	*Area 107*	*Area 110*	*Area 112*	*Area 118*	*Area 121*	*Area 123*	*Area 134*	*Area 159*	*Area 167*	*Area 171*	*Area 176*	*Area 178*	*Area 185*	*Area 186*
UC Hotel	Coeff	1.87E-05	-6.25E-05	9.71E-05	2.94E-04	-3.68E-05	-3.14E-05	4.80E-05	-2.31E-04	4.61E-05	9.30E-05	-8.92E-04	2.49E-03	-8.21E-05	-2.51E-05	-5.28E-05	2.98E-05	-3.65E-05
	Std Err	(4.07E-06)	(4.32E-05)	(3.25E-05)	(3.91E-04)	(4.43E-05)	(3.78E-05)	(2.15E-05)	(1.58E-04)	(2.84E-05)	(4.28E-05)	(4.16E-04)	(1.08E-03)	(4.74E-05)	(1.37E-04)	(5.31E-05)	(3.72E-05)	(3.85E-05)
Area 105	Coeff	1.18E-03	1.57E-02	5.27E-05	-5.23E-04	4.97E-04	-1.65E-04	8.28E-04	1.04E-03	1.30E-04	-2.63E-05	2.62E-04	-2.69E-03	-1.62E-04	-6.95E-04	1.29E-03	-1.28E-04	1.57E-04
	Std Err	(5.75E-04)	(5.21E-03)	(2.86E-04)	(4.32E-04)	(2.10E-03)	(5.70E-04)	(7.07E-04)	(5.03E-04)	(6.31E-04)	(5.41E-05)	(5.53E-03)	(1.44E-02)	(5.13E-04)	(1.82E-03)	(4.96E-04)	(3.78E-04)	(5.90E-04)
Area 106	Coeff	6.76E-06	2.12E-04	-1.81E-04	-6.31E-03	8.32E-04	-1.16E-04	1.29E-05	-1.20E-04	-5.77E-04	6.62E-05	6.34E-04	6.35E-03	1.85E-04	1.74E-03	1.78E-04	-1.24E-04	-9.00E-05
	Std Err	(1.97E-04)	(2.62E-04)	(1.30E-04)	(2.37E-03)	(9.55E-04)	(2.59E-04)	(3.22E-04)	(2.29E-04)	(2.87E-04)	(2.46E-05)	(2.52E-03)	(6.54E-03)	(2.34E-04)	(8.30E-04)	(2.26E-04)	(1.72E-04)	(2.69E-04)
Area 107	Coeff	1.86E-03	-1.02E-04	-9.60E-05	1.19E-04	-8.42E-04	-9.53E-06	6.71E-05	5.13E-04	9.46E-05	5.21E-05	-6.20E-03	5.11E-03	-2.43E-04	1.34E-03	5.64E-05	1.08E-04	-2.43E-04
	Std Err	(2.88E-03)	(3.18E-04)	(1.58E-04)	(2.39E-04)	(1.16E-03)	(3.15E-04)	(3.91E-04)	(2.78E-04)	(3.49E-04)	(2.99E-05)	(3.06E-03)	(7.95E-03)	(2.84E-04)	(1.01E-03)	(2.74E-04)	(2.09E-04)	(3.27E-04)
Area 110	Coeff	-8.24E-05	-2.01E-04	2.28E-04	-3.90E-03	-2.72E-04	1.70E-03	-2.10E-04	-4.59E-04	-1.53E-04	2.67E-05	6.39E-05	8.14E-03	-1.98E-04	2.52E-04	5.01E-05	6.09E-04	1.58E-04
	Std Err	(3.12E-04)	(3.04E-04)	(2.29E-04)	(2.76E-03)	(1.11E-03)	(3.01E-04)	(1.51E-04)	(2.66E-04)	(2.00E-04)	(2.86E-05)	(2.93E-03)	(7.61E-03)	(3.34E-04)	(9.65E-04)	(3.74E-04)	(2.62E-04)	(2.71E-04)
Area 112	Coeff	3.33E-03	-6.28E-05	1.06E-03	8.09E-03	-3.55E-04	2.09E-04	-2.33E-04	2.10E-03	-3.65E-04	-2.32E-05	-4.80E-03	8.64E-03	-4.72E-04	1.73E-03	8.51E-04	8.16E-04	5.54E-05
	Std Err	(5.99E-04)	(6.85E-04)	(5.15E-04)	(6.20E-03)	(7.03E-04)	(6.78E-04)	(3.40E-04)	(2.50E-03)	(4.50E-04)	(6.44E-05)	(6.59E-03)	(1.71E-02)	(7.52E-04)	(2.17E-03)	(8.42E-04)	(5.90E-04)	(6.11E-04)
Area 118	Coeff	1.46E-03	-5.52E-04	-7.13E-05	1.33E-02	2.60E-04	1.30E-04	6.83E-04	-2.18E-04	3.12E-05	1.39E-05	5.41E-03	4.82E-03	-2.20E-05	-3.66E-04	-4.85E-04	5.49E-04	1.38E-03
	Std Err	(2.49E-04)	(5.01E-04)	(3.76E-04)	(4.54E-03)	(1.83E-03)	(4.96E-04)	(6.16E-04)	(4.38E-04)	(5.50E-04)	(4.71E-05)	(4.82E-03)	(1.25E-02)	(4.47E-04)	(1.59E-03)	(4.32E-04)	(3.29E-04)	(5.14E-04)
Area 121	Coeff	5.78E-04	-2.54E-04	1.98E-04	-1.73E-03	4.59E-04	4.19E-05	-2.02E-04	2.55E-04	3.83E-05	5.34E-05	-1.37E-03	-3.42E-03	-4.40E-04	2.69E-04	3.48E-04	2.90E-04	4.28E-04
	Std Err	(1.06E-03)	(2.92E-04)	(2.19E-04)	(2.64E-03)	(2.99E-04)	(2.89E-04)	(1.45E-04)	(2.55E-04)	(1.91E-04)	(2.74E-05)	(2.81E-03)	(7.29E-03)	(3.20E-04)	(9.25E-04)	(3.58E-04)	(2.51E-04)	(2.60E-04)
Area 123	Coeff	5.33E-04	1.87E-04	7.75E-05	2.74E-03	2.51E-04	-1.64E-04	9.01E-06	-3.19E-05	-3.92E-04	3.86E-05	4.11E-03	-8.11E-03	5.56E-04	7.24E-04	1.84E-04	-2.38E-04	6.55E-04
	Std Err	(1.46E-04)	(2.23E-04)	(1.68E-04)	(2.02E-03)	(8.14E-04)	(2.21E-04)	(1.11E-04)	(1.95E-04)	(2.45E-04)	(2.10E-05)	(2.14E-03)	(5.57E-03)	(1.99E-04)	(7.07E-04)	(2.74E-04)	(1.92E-04)	(2.29E-04)
Area 134	Coeff	1.42E-03	5.22E-05	5.18E-06	-2.67E-03	-3.25E-04	-2.29E-04	1.16E-04	7.31E-04	-1.87E-04	2.69E-05	-1.09E-03	-9.52E-04	-1.89E-04	3.29E-04	-3.50E-04	2.72E-04	1.01E-04
	Std Err	(2.28E-04)	(2.30E-04)	(1.73E-04)	(2.08E-03)	(2.36E-04)	(2.01E-04)	(1.14E-04)	(8.38E-04)	(1.51E-04)	(2.16E-05)	(2.21E-03)	(5.74E-03)	(2.52E-04)	(7.29E-04)	(2.82E-04)	(1.98E-04)	(2.05E-04)
Area 159	Coeff	-4.59E-03	6.19E-03	-4.04E-04	-9.98E-04	2.50E-06	5.01E-04	1.35E-03	-2.18E-04	-2.70E-04	4.39E-05	-1.53E-02	-1.31E-03	1.58E-03	-5.53E-04	-8.98E-04	1.36E-03	4.85E-03
	Std Err	(1.08E-02)	(1.01E-02)	(5.57E-04)	(8.42E-04)	(4.09E-03)	(1.11E-03)	(1.38E-03)	(9.80E-04)	(1.23E-03)	(1.05E-04)	(2.80E-02)	(3.55E-03)	(9.99E-04)	(1.12E-03)	(9.66E-04)	(7.35E-04)	(1.15E-03)
Area 167	Coeff	-6.09E-03	-7.38E-03	3.21E-04	2.98E-04	1.44E-03	2.76E-04	6.80E-04	2.96E-04	6.27E-05	-1.80E-05	6.38E-03	2.20E-03	4.22E-04	2.79E-04	-1.15E-04	2.19E-05	-3.87E-04
	Std Err	(7.38E-03)	(2.67E-03)	(1.47E-04)	(2.22E-04)	(1.08E-03)	(2.92E-04)	(3.63E-04)	(2.58E-04)	(3.24E-04)	(2.78E-05)	(2.84E-03)	(9.36E-04)	(2.63E-04)	(2.95E-04)	(2.54E-04)	(1.94E-04)	(3.03E-04)
Area 171	Coeff	-1.41E-04	2.43E-05	-1.74E-04	-1.29E-03	-7.39E-04	-7.46E-05	4.98E-05	2.49E-05	8.36E-05	2.68E-05	-4.80E-04	3.73E-03	9.23E-05	1.05E-03	4.70E-06	-8.84E-06	3.79E-04
	Std Err	(2.11E-04)	(1.92E-04)	(1.44E-04)	(1.74E-03)	(7.00E-04)	(1.90E-04)	(9.53E-05)	(1.68E-04)	(1.26E-04)	(1.81E-05)	(1.85E-03)	(4.80E-03)	(1.71E-04)	(6.09E-04)	(2.36E-04)	(1.65E-04)	(1.97E-04)
Area 176	Coeff	-5.36E-04	-1.68E-02	-2.75E-04	1.97E-03	7.29E-03	9.13E-05	-3.93E-04	5.06E-04	-2.01E-03	1.76E-04	-7.03E-03	1.56E-02	4.91E-04	-2.65E-04	1.50E-03	6.58E-05	-2.02E-04
	Std Err	(2.43E-03)	(6.94E-03)	(3.81E-04)	(5.76E-04)	(2.80E-03)	(7.59E-04)	(9.42E-04)	(6.71E-04)	(8.41E-04)	(7.21E-05)	(7.37E-03)	(1.92E-02)	(6.84E-04)	(7.66E-04)	(6.61E-04)	(5.03E-04)	(7.87E-04)
Area 178	Coeff	-3.97E-04	-1.55E-06	5.36E-05	4.40E-03	-1.19E-03	-7.12E-05	1.22E-03	-4.27E-04	5.06E-04	5.16E-05	-6.09E-03	-7.38E-03	-1.34E-04	6.32E-04	1.45E-04	-2.36E-04	-8.02E-05
	Std Err	(6.82E-04)	(5.55E-04)	(4.17E-04)	(5.03E-03)	(2.03E-03)	(5.50E-04)	(2.76E-04)	(4.86E-04)	(6.09E-04)	(5.22E-05)	(5.34E-03)	(1.39E-02)	(4.95E-04)	(1.76E-03)	(4.79E-04)	(3.64E-04)	(5.70E-04)
Area 185	Coeff	7.89E-04	4.97E-05	3.92E-04	-3.46E-03	1.46E-03	-1.10E-04	-8.63E-05	4.16E-04	-3.24E-04	-3.16E-06	1.28E-04	-9.81E-05	7.24E-05	-2.65E-04	6.40E-04	2.76E-04	9.74E-04
	Std Err	(2.80E-04)	(3.25E-04)	(2.44E-04)	(2.95E-03)	(1.19E-03)	(3.22E-04)	(1.62E-04)	(2.85E-04)	(3.57E-04)	(3.06E-05)	(3.13E-03)	(8.13E-03)	(2.90E-04)	(1.03E-03)	(4.00E-04)	(2.14E-04)	(3.34E-04)
Area 186	Coeff	1.31E-04	1.53E-04	2.17E-05	-1.51E-04	-1.69E-04	-1.27E-04	2.84E-05	-3.64E-05	-1.98E-05	2.03E-05	-1.49E-04	5.10E-04	-1.05E-04	9.61E-05	-9.24E-05	-1.07E-05	2.05E-05
	Std Err	(1.49E-04)	(1.67E-04)	(1.26E-04)	(1.52E-03)	(6.11E-04)	(1.66E-04)	(8.32E-05)	(1.47E-04)	(1.10E-04)	(1.58E-05)	(1.61E-03)	(4.19E-03)	(1.84E-04)	(5.31E-04)	(2.06E-04)	(1.44E-04)	(1.72E-04)

Appendix C: Statistical Properties of Empirical Inverse Jacobian

This appendix addresses the sampling properties the Empirical Inverse Jacobian specified in the regression equation (7.5):

$$\begin{bmatrix} \Delta p_{1t} \\ \vdots \\ \Delta p_{Nt} \end{bmatrix} = \Delta p_t = Bz_t\left(p_{t-1}\right) + \varepsilon_t = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1N} \\ \vdots & \ddots & \vdots \\ \beta_{N1} & \cdots & \beta_{NN} \end{bmatrix} \begin{bmatrix} z_{1t} \\ \vdots \\ z_{Nt} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \vdots \\ \varepsilon_{Nt} \end{bmatrix} \tag{C.1}$$

The sampling properties of this regression equation are straightforwardly derived within a SUR framework. Under the usual regularity conditions, the estimated coefficients in the B matrix are consistent and asymptotically normally distributed. Given this asymptotic normality for the coefficients in the B matrix, testing the two conditions necessary for stability of the Empirical Inverse Jacobian is a relatively straightforward exercise.

C.1 Asymptotic Distribution and Testing Hypotheses for Individual Coefficients

The first condition for stability of the Empirical Inverse Jacobian holds that the diagonal entries in the B matrix are non-negative, setting up the null hypothesis that these coefficients are negative against the alternative that they are weakly positive. The usual test statistics for evaluating this null hypothesis can be estimated using the asymptotic approximation and asymptotic standard errors without complication as presented in section 7.1.

C.2 Asymptotic Distribution and Testing Hypotheses for Determinants of Principal Minors

The second condition for stability of the Empirical Inverse Jacobian holds that the determinants of each principal minor are weakly negative. Denoting the principal minor for the m^{th} market by

$$B_{(-m)} = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1,m-1} & \beta_{1,m+1} & \cdots & \beta_{1,N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \beta_{m-1,1} & \cdots & \beta_{m-1,m-1} & \beta_{m-1,m+1} & \cdots & \beta_{m-1,N} \\ \beta_{m+1,1} & \cdots & \beta_{m+1,m-1} & \beta_{m+1,m+1} & \cdots & \beta_{m+1,N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \beta_{N,1} & \cdots & \beta_{N,m-1} & \beta_{N,m+1} & \cdots & \beta_{N,N} \end{bmatrix}.$$

Letting $\tau_m \equiv \det\left(B_{(-m)}\right)$ denote the determinant for this principal minor matrix, we seek to test the hypothesis:

$$H_0 : \tau_m \leq 0 \text{ versus } H_1 : \tau_m > 0 .$$

Note that the estimated coefficients are all jointly asymptotically normally distributed and the determinant is a continuous function. As such, the estimated determinant of the principal minor, which replaces the entries of $B_{(-m)}$ with their OLS estimates (denoted $\hat{B}_{(-m)}$), is also consistent and asymptotically normal:

$$\hat{\tau}_m \equiv \det\left(\hat{B}_{(-m)}\right) \to N\left(\tau_m, \sigma^2_{\tau_m}\right)$$

The asymptotic variance $\sigma^2_{\tau_m}$ can, in principal, be found using a delta-method approximation. In practice, though, such an analytical exercise would prove exceedingly complicated. In practice, this asymptotic variance can be approximated using numerical methods applying a bootstrap technique.

We adopt a parametric bootstrap for to estimate the variance $\sigma^2_{\tau_m}$. The bootstrap algorithm we adopt, and our methodology for constructing confidence intervals, proceeds as follows:

Algorithm 1: Bootstrap Confidence Intervals for Principal Minor Determinants

Step 1: Estimate the regression model C.1

Step 2: Recover the estimated coefficients $\hat{B}$ and residuals $\left\{\hat{\varepsilon}_t\right\}_{t=1}^{T}$, where, $\hat{\varepsilon}_t = \left[\hat{\varepsilon}_{1t}, \ldots, \hat{\varepsilon}_{Nt}\right]'$

Step 3: Estimate the covariance matrix for residuals from $\left\{\hat{\varepsilon}_t\right\}_{t=1}^{T}$ applying the usual degree of freedom correction, denoted $\hat{\Sigma}_{\varepsilon}$.

Step 4: Generate K Bootstrap Samples:

For $k = 1, \ldots, K$

Step k.1: Draw simulated residuals $\left\{\tilde{\varepsilon}_t^{(k)}\right\}_{t=1}^{T}$, where, $\tilde{\varepsilon}_t^{(k)} \sim N\left(0,\ \hat{\Sigma}_{\varepsilon}\right)$

Step k.2: Calculate $\left\{\Delta p_t^{(k)}\right\}_{t=1}^{T}$, where, $\Delta p_t^{(k)} = \hat{B} z_t\left(p_{t-1}\right) + \tilde{\varepsilon}_t^{(k)}$

Step k.3: Estimate regression model: $\Delta p_t^{(k)} = \tilde{B}^{(k)} z_t\left(p_{t-1}\right) + u_t^{(k)}$

Step k.4: Compute the principal minor determinants, $\tau_m^{(k)} = \det\left(\tilde{B}_{(-m)}^{(k)}\right)$

Next k

Step 5: Compute the bootstrap standard error, $\hat{\sigma}^2_{\tau_m}$, for the m^{th} principal minor determinant from the sample $\left\{\tau_m^{(k)}\right\}_{k=1}^{K}$.

Step 6: Compute the 95% confidence interval using the usual critical values:

$$\hat{C}_{0.95}\left(\hat{\tau}_m\right) = \left[\hat{\tau}_m - 1.96\hat{\sigma}_{\tau_m},\ \hat{\tau}_m + 1.96\hat{\sigma}_{\tau_m}\right]$$

Applying this algorithm for each of the markets under consideration provides a straightforward mechanism for calculating the confidence intervals reported in Table 7.3, Panel B.

C.3 Testing Hypotheses for Maximum Determinants of All Principal Minors

The restriction that the determinants of all principal minors are weakly negative can be analyzed by considering the maximum of the determinants for each of the principal minors considered in the previous subsection. Specifically, define:

$$\tau = \max_{m=1,\dots,N} \left\{ \tau_m \right\}$$

As the max operator is another continuous function, estimates for τ satisfy all of the asymptotic sampling properties from the previous section, so that:

$$\hat{\tau} \equiv \max_{m=1,\dots,N} \left\{ \hat{\tau}_m \right\} \to N\left(\tau, \sigma_\tau^2\right)$$

As above, the asymptotic variance can in principal be solved for analytically, but in practice can be estimated using the bootstrap samples generated from Algorithm 1.

We begin by applying Algorithm 1, Steps 1 through 3. We then perform Algorithm 1, Step 4 for $m = 1, \dots, N$. We generate the bootstrap sample $\left\{\tau^{(k)}\right\}_{k=1}^{K}$, with $\tau^{(k)} = \max_{m=1,\dots,N} \left\{\tau_m^{(k)}\right\}$, and use this sample to calculate the bootstrap mean $\hat{\tau}$ and bootstrap variance $\hat{\sigma}_\tau^2$. Given these estimates, which are consistent for the population parameters, we can then construct the 95% confidence interval as:

$$\hat{C}_{0.95}\left(\hat{\tau}\right) = \left[\hat{\tau} - 1.96\hat{\sigma}_\tau, \hat{\tau} + 1.96\hat{\sigma}_\tau\right]$$

This confidence interval is reported in Table 7.3, Panel A.