DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA 91125

Divergence and Convergence in Scarf Cycle Environments: Experiments and Predictability in the Dynamics of General Equilibrium Systems

Benjamin J. Gillen
California Institute of Technology

Masayoshi Hirota
Information and Social Science Research

Ming Hsu
University of California, Berkeley

Charles R. Plott
California Institute of Technology

Brian W. Rogers
Washington University St. Louis

SOCIAL SCIENCE WORKING PAPER 1239

March, 2018

ABSTRACT
Divergence and Convergence in Scarf Cycle Environments: Experiments and Predictability in the Dynamics of General Equilibrium Systems[1]

Benjamin J. Gillen
California Institute of Technology

Masayoshi Hirota
Information and Social Science Research

Ming Hsu
University of California Berkeley

Charles R. Plott
California Institute of Technology

Brian W. Rogers
Washington University Saint Lewis

ABSTRACT

Previous experimental work demonstrates the power of classical theories of economic dynamics to accurately predict major features of price dynamics in multiple market systems. Building on this literature, this study implements experimental markets designed after extreme environments identified by Scarf (1960) and Hirota (1981). These environments provide insight into two important economic questions: (a) do markets necessarily converge to a unique interior equilibrium? and (b) which model, among a set of classical specifications, most accurately characterizes observed price dynamics? Our first result demonstrates that the dynamic property of "expanding price orbits" exists, with prices spiraling outwardly around the equilibrium prices in the directions predicted by the theory of disequilibrium price dynamics. Our second result establishes properties of partial equilibrium theory in an unstable general equilibrium environment. Price changes in a market reflect the magnitude of excess demand of that market, with excess demand in other markets making second-order contributions to predicted price changes. These results support the fundamental principle, advanced by Walras and others, that the direction of price change in a given market depends only on the sign of its own excess demand. This excess demand may depend on many prices, but unless disequilibrium is severe the sign of the price change does not depend on the magnitude of excess demand in other markets.

[1] The financial support of the John Templeton Foundation and the Caltech Laboratory for Experimental Economics and Political Science are gratefully acknowledged. Comments of Peter Bossaerts, Anjan Mukherji and Bill Zame contributed significantly.

1. Introduction

The paper studies and experimentally confirms the existence of cyclical and exploding price dynamics predicted by the theoretical works of Scarf (1960) and Hirota (1981). In doing so, the study lends support to the classical principles underlying models characterizing multi-market dynamics. Previous experimental work demonstrates robust convergence of prices to the competitive equilibrium, possibly punctuated by aspects of local instability and bubble-like price patterns. Many of these studies explored theoretically well-behaved environments in which the predictions of equilibrium analysis and multi-market price dynamics stand broadly in accord. Predictions of unbounded divergence and the possibility of perpetual price movement suggested by Scarf and Hirota's model of price dynamics, which stand in sharp contrast to the predictions of equilibrium, had not yet been explored in the laboratory setting. In so doing, we identify the underlying principles driving price dynamics as devices that underscore the positive value of general equilibrium theory for understanding market behavior.

The Scarf (1960) and Hirota (1981) models consider price formation from the abstract perspective of a Walrasian auctioneer capable of measuring excess demand at disequilibrium prices without implementing a market system in which trades take place at such prices. While this abstraction is well-suited to forming logical conceptualizations of market adjustment processes, it also presents a major departure from the procedural details of how actual markets function and the strategic behavior of individuals participating in markets. Experimental markets, by contrast, implement actual trading institutions and features of price discovery in which there is no fictional Walrasian auctioneer. Instead, bids and asks are tendered by potential traders themselves in real time and, as trades take place at different prices, demand and supply curves shift as trading proceeds. Vernon Smith (1965)'s discovery of price convergence in market experiments demonstrates the close connection between a theory derived from abstractions and the data drawn from a completely different environment. The abstractly formulated theory of price processes generates predictive power even when applied to a very different environment subjected to a host of frictions assumed away by the theoretical abstraction. Many studies of experimental markets show that markets tend to "equilibrate" at a pattern of prices and allocations that are near the equilibrium of the fictional Walrasian auctioneer.[2]

[2] The role abstract, axiomatic principles of (multiple) market behavior which address the details of neither market institutions nor individual decisions, have been a topic of much discussion in the economics literature. The experimental markets converge to the equilibrium predictions of a model that clearly lacks descriptive accuracy. While the markets follow elements of "as if" the Walrasian auction was in control the precise reasons for the accuracy of the model is, as Smith asserts, a mystery.

Experiments lend support for both the principles of equilibrium and excess demand that support the underlying dynamic model of price formation in the classical competitive model. Convergence of the continuous double auction toward the competitive equilibrium prices and allocation is a reliable property of markets under a wide range of environments. Unstable equilibria, price bubbles and cyclic price movements have all been observed and studied as features of price movements.[3] The Scarf (1960) and Hirota (1981) models present a theoretical abstraction not yet explored in experimental markets and predict a broader set of price dynamics, including the possibility that the Walrasian auctioneer's algorithm for price discovery need not converge. Indeed, the model suggests that a price discovery mechanism may settle into a limit cycle following a very slowly expanding orbit.

This paper explores these possibilities and thus provides an opportunity to better understand subtle features of market adjustment by building on a host of developments in the design and implementation of experimental markets. Section 2 of the paper provides some background on the experimental foundations of the design and the theoretical basis of the study. Section 3 details the experimental setting, including agent incentives, model predictions, and many of the practical elements of the implemented market design.

Section 4 discusses the observed price dynamics, presenting suggestive evidence in support of the key model prediction that prices fail to converge to the theoretical equilibrium. Observed prices spiral around the equilibrium as predicted by models of excess demand before eventually hitting a price boundary, well away from equilibrium. We study this feature statistically in Section 5, demonstrating that (a) prices trend away from the theoretical equilibrium, (b) the random price changes are only weakly attracted to theoretical equilibrium, (c) excess demands do not converge to zero, and (d) gains from trade persist despite the cessation of trading activity. Though these results are inconsistent with static equilibrium analysis, they match the predicted dynamics implied by classical models of excess demand dynamics.

Having established equilibrium non-convergence, we estimate several structural models of price dynamics proposed in the literature in section 6. Our analysis not only evaluates

[3] The phenomenon of market instability is first observed in Plott and George (1992) for the case of a Marshallian externality. It is replicated by Plott and Smith (1999). Instability in the case of income effects, is first observed in Plott (2000). Cyclical price patterns in Scarf-type environments with slow convergence are suggested in Anderson, et.al. (2004). Good reviews of the literature are found in Plott and Smith (2008). Clearly, the details of the market organization can have a profound effect on both behavior and competing theories. For example, Gintis (2007) presents simulations demonstrating equilibration in a Scarf-type environment when prices are only privately known and thus uncoordinated. Indeed, Gintis (2007) conjectures that a public price is a coordinating device that creates the instability. Plott and Pogorelskiy (2017) find evidence of dynamic convergence in an exchange call markets similar to a Newton Method of price determination.

the relative significance of excess demand and prices as drivers of observed prices, but also allows insight into the relative importance of partial equilibrium and general equilibrium forces in determining prices and allocations. We find that, while general equilibrium forces are non-negligible, their influence on prices is second-order relative to the partial equilibrium adjustments of excess demand and disequilibrium in its own market. These results suggest a fundamental principle from Walras and others that the direction of price change in a given market depends only on the sign of its own excess demand is only violated under conditions of extreme disequilibrium, possibly through expectations of future prices in interdependent markets.

2. Background

Experimental methods in economics evolved as tools to create simple and special case markets in which the broad, abstract and general principles of economics can be studied under controlled conditions. The key elements are the commodity space, the preferences and the trading institutions, all of which support the creation of a simple market system to which general economic principles apply. While the experimental markets are simple special cases of markets, they are nevertheless, real markets. Though simplicity should not be confused with reality, general principles are expected to apply even in the simple and special cases.

The methods rest on the creation of a commodity space and the use of money to induce preferences over the commodities that can be traded. Time exists as a "period" in which trading of money and commodities take place in real time with the benefits of trading in terms of money earned are realized at the end of a period. The experiment proceeds as multiple periods or trading days that could be interpreted as a week. In a stationary environment, the periods are identical except for the information and benefits gained from previous periods. Trading takes place in a market organized by an architecture of institutions known to support efficient trading. Models are applied with an "as if" methodology with trading within a period and over periods both studied with equilibration predicted by models expected after multiple periods.

A. Market Architecture

The markets were conducted as a continuous, multiple unit double auction, MUDA, introduced by Plott and Grey (1990) through an electronic market place developed by the Caltech Laboratory for Experimental Economics and Political Science (EEPS) called Marketscape. This market platform supports multiple, simultaneous, continuous markets. The markets are open for a fixed time called a period similar to a trading day.

When a period opens traders are free to tender bids to buy as a price per unit and maximum quantity or ask to sell units (a price per unit and maximum quantity). The markets has open (public) books that record bids and asks whenever they do not automatically trigger a trade by matching an already existing bid or ask in the order book. The bids are exposed to the market with price priority from highest price to lowest, while asks are exposed with price priority from lowest to highest, with ties prioritized by the time at which the bid or ask was entered. When a trade executes, the transaction is immediately recorded and units of inventory and money are instantaneously transferred between trading parties. Orders may be partially filled, with any unfilled portion remaining on the order book. Bids and asks remain in the book throughout a period unless expired, cancelled, or executed in a trade. In addition to the order book, all participants are able to view all data from all trades in continuous time through either a periodically updated graph or a listing of executed trades.

When a period closes subjects acquire the money they made, based on their end-of-period holdings according to induced preferences (described in section 3A), from the contracts they developed during the period. Upon the close of a period, the system validates accounting to record profits earned by participants based on the end-of-period holdings. Subjects' inventories are then reset to their initial endowments, the order book is cleared, and a new period of trading opens. Since stocks cannot be traded across periods, each trading period can be analyzed as a single market realization. However, the periods within a given session are not independent due to substantial price persistence from the end of one period to the beginning of the next.

We note that the continuous double auction institution involves trading executed at disequilibrium prices and, over the course of each period, at a variety of such prices. The markets produce two different time series of contract prices. First, "instantaneous prices" consist of the contracts that take place within a period. In experiments with one commodity instantaneous prices within a period typically exhibit erratic movement towards the "competitive equilibrium price"[4]. Second, "period prices" record the evolution of prices across periods as summarized by the quantity-weighted average price within a period. Findings from other experiments suggest two typical patterns. First, instantaneous prices tend to converge to near the competitive price. Second, period prices also converge across periods with the Plott and Smith (1978) efficiency measure

[4] From an assumption that a constant market price exists in the market, as if called out by a "Walrasian auctioneer", the redemption values and costs can be used to compute a market demand function and a market supply function from which a single, market clearing price can be computed. Convention has named this price the "competitive equilibrium price".

approaching 100% after several periods.[5] The efficiency measures suggest that gains from trade become exhausted, which is sometimes viewed as a form of equilibrium.

B. The Classical Model

Consider a setting with two commodities X_1 and X_2 and prices P_1 and P_2 and denote the excess demands for goods X_1 and X_2 by $Z_1\left(P_1, P_2\right)$ and $Z_2\left(P_1, P_2\right)$, respectively. The classical competitive equilibrium model defines the theoretical equilibrium to be the prices at which both $Z_1\left(P_1, P_2\right) = 0$ and $Z_2\left(P_1, P_2\right) = 0$. In that context, this equilibrium is completely static with final prices and allocations based on a fixed demand and supply. However, the equations are used as tools to explore the forces guiding price discovery with a dynamic interpretation.

We define the "Classical Model" of dynamics evolving from Walras' fundamental principle that prices respond to excess demand in a good's own market. Under this model, the change in prices for goods X_1 and X_2 ($\dot{P}_1$ and $\dot{P}_2$, respectively) scale linearly with the excess demand for each respective good, so that

$$\dot{P}_1 = a_{11} Z_1\left(P_1, P_2\right), \text{ and, } \dot{P}_2 = a_{22} Z_2\left(P_1, P_2\right). \tag{1}$$

The parameters a_{11} and a_{22} reflect the relative speed with which a market price accommodates, or adjusts to, its excess demand. These parameters can play a central role in characterizing models of dynamics and stability. Hicks, for example, develops a model in which markets adjust at different rates as central feature of his model of partial and general equilibration.[6]

One could imagine alternative adjustment processes characterizing price dynamics and a substantial literature explores the properties of such alternatives. We will discuss such alternatives later, along with useful empirical generalizations, in Section 7. At this point, the Classical Model provides sufficient structure to motivate the theory underlying the current study's design.

---

[5] The concept of efficiency was developed by Plott and Smith (1978). "Social benefits" are typically defined as sum of the redemption values of buyers from the contracts of which they are a part and the "social cost" are the cost to sellers of supplying those units. Efficiency is the actual difference between social benefits and cost in a period divided by the maximum possible given the redemption values of buyers and cost of the sellers.

[6] Hicks posed a question about the relationship between partial equilibrium and stability. McFadden (1969) formalizes a concept of partial equilibrium demonstrates a close connection between Hick's condition for partial equilibrium and Samuelsonian models of stability.

3. Experimental Environment

Specifying the mechanism for implementing markets in this study required many operational decisions to ensure a feasible experimental protocol while preserving essential elements of the general equilibrium setting we seek to study. To this end, we implement two markets for the commodities X_1 and X_2 featuring continuous double auctions with a limit-order book where prices of a single unit of X_1 and X_2 are quoted in terms of the number of units X_3. This implementation places commodity X_3 as the numeraire, allowing us to plot prices of commodities X_1 and X_2 in terms of units of X_3.[7]

A. Preferences and Endowments

Our experiments induced subjects' preferences to be similar to those studied theoretically by Scarf (1960) and Hirota (1981) and experimentally by Anderson, et.al. (2004), and Plott (2001), with substantial modifications. Agents have perfectly complementary Leontief preferences for two of the commodities while deriving no utility from the third. By design, these preferences and initial endowments ensure the existence of a unique, interior competitive equilibrium for all experiments. However, the existence of this equilibrium need not imply observed transactions occur at equilibrium prices.

Table 1: Preferences, Endowments, and Equilibrium Allocations (X_{1i}, X_{2i}, X_{3i})

Clockwise	Type 1	Type 2	Type 3
Utility	$70 \min\left\{\frac{3X_{2i}}{4}, \frac{X_{3i}}{80}\right\}$	$70 \min\left\{\frac{X_{1i}}{2}, \frac{3X_{3i}}{80}\right\}$	$70 \min\left\{\frac{3X_{1i}}{2}, \frac{X_{2i}}{4}\right\}$
Endowment	(0, 0, 800)	(20, 0, 0)	(0, 40, 0)
Equilibrium	(0, 10, 600)	(15, 0, 200)	(5, 30, 0)
Counterclockwise			
Utility	$210 \min\left\{\frac{X_{2i}}{12}, \frac{X_{3i}}{80}\right\}$	$210 \min\left\{\frac{X_{1i}}{2}, \frac{X_{3i}}{240}\right\}$	$210 \min\left\{\frac{X_{1i}}{6}, \frac{X_{2i}}{4}\right\}$
Endowment	(0, 40, 0)	(0, 0, 800)	(20, 0, 0)
Equilibrium	(0, 30, 200)	(5, 0, 600)	(15, 10, 0)

We selected preferences and initial endowments so the classical model captured by equation (1) predicts prices diverge from the competitive equilibrium given any non-

[7] In all of our specifications, endowments of commodity X_3 are much greater than the endowments of the other two commodities. Since prices are in terms of units of X_3, finely divisible units of X_3 must exist to prevent the integer constraint from substantially reducing the number of feasible prices.

equilibrium initial price vector and do so in a predictable fashion.[8] As such, the unique, interior, competitive equilibrium is unstable according to this classical model. The nature of this instability depends on the preference parameters and endowments that we explore in two opposing specifications, the "Clockwise" and "Counterclockwise" treatments.[9] Table 1 presents the specific magnitudes of the utility parameters and initial endowments, with Figure 1 illustrating indifference curves for each type in the Clockwise treatment.

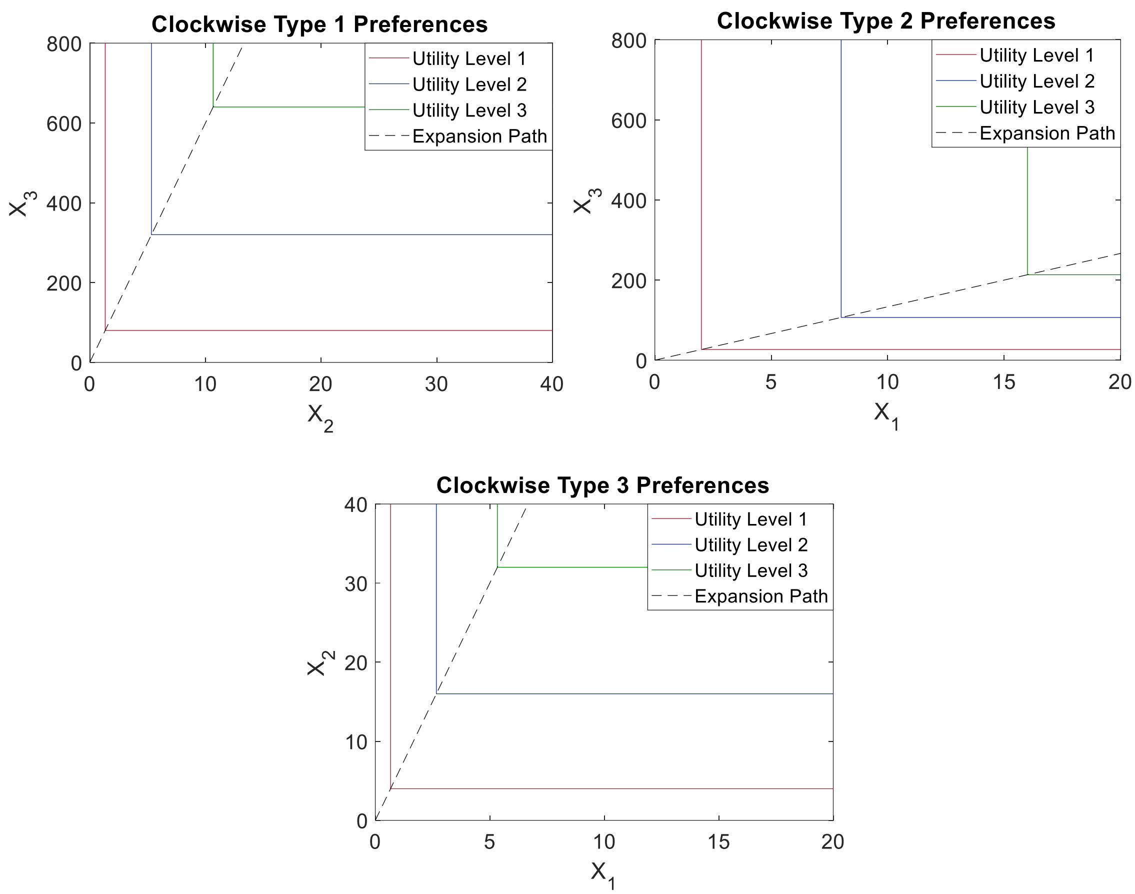

Figure 1: Indifference Curves for Each Clockwise Treatment Type

[8] As will be discussed later, the prediction of the classical model depends importantly upon the preference parameters and the initial endowments. See Appendix A for a discussion of the general class of models from which the experimental parameters were chosen.

[9] Previewing results from the next section, the classical model predicts prices under the clockwise (counterclockwise) treatment will diverge in a clockwise (counterclockwise) direction when plotted in the two-dimensional price space for Commodities X_1 and X_2, treating X_3 as the numeraire.

B. Competitive Equilibrium and Excess-Demand Driven Classical Dynamics

Given agents' preferences and endowments, we can solve for the excess demand equations for the exchange economy.[10] Letting $P = (P_1, P_2)'$ denote the prices for X_1 and X_2, Table 2 presents the market excess demand function at initial endowments when each type of agent is present in equal proportion. Solving for equilibrium prices by setting the equations in Table 2 to zero with X_3 as a numeraire, yields the theoretical equilibrium prices for X_1 and X_2 equal to 40 and 20 in both the clockwise and counterclockwise treatments. The implied allocations under this equilibrium appear alongside the endowments in Table 1.

Table 2: Market Excess Demand for Commodities X_1 and X_2 at Initial Endowments

	$Z_1(P_1, P_2)$	$Z_2(P_1, P_2)$
Clockwise	$\frac{60P_1}{40+3P_1}+\frac{40P_2}{P_1+6P_2}-20$	$\frac{800}{60+P_2}+\frac{240P_2}{P_1+6P_2}-40$
Counterclockwise	$\frac{800}{120+P_1}+\frac{60P_1}{3P_1+2P_2}-20$	$\frac{40P_1}{3P_1+2P_2}+\frac{120P_2}{20+3P_2}-40$

The experimental design is inspired by the theoretical literature on classical principles of dynamic adjustment.[11] The market specifications from Scarf (1960) and Hirota (1981) provide interesting settings in which classical forces do not guide prices to converge toward the competitive equilibrium. As presented in Figure 2, a simple phase diagram characterizes the dynamic behavior of the "Classical Model" in which $\dot{P}_1 \propto Z_1(P_1, P_2)$ and $\dot{P}_2 \propto Z_2(P_1, P_2)$. The partial equilibrium curve $Z_1(P_1, P_2) = 0$ represents the pairs of prices for which the excess demand of X_1 is zero regardless of excess demand for X_2, with the curve $Z_2(P_1, P_2) = 0$ defined analogously. These curves intersect at the theoretical equilibrium prices $P^* = [40, 20]$, with zero excess demand for both goods.

The partial equilibrium curve for commodity X_1 divides the price space into regions for which the excess demand for X_1 is positive (negative), placing upward (downward) price pressure on X_1. The partial equilibrium curve for commodity X_2 similarly divides the

[10] Assuming perfectly liquid markets in which agents' behave as price-takers allow us to specify their demand (or supply) of each commodity considering only their budget constraint and initial endowments. Summing these individual demand functions and subtracting the total endowments of each commodity characterizes the market excess demand function. Equilibrium then attains when excess demand functions equal zero and market demand equals market supply for each commodity.

[11] For expositional purposes, we defer a deeper discussion of this literature to section 6, when we evaluate how well different models describe the observed data. Though our market implementation deviates from the frictionless assumptions imposed by the price adjustment processes derived in much of this literature, the design is driven by predictions from these classical models of dynamics.

space into areas in which the excess demand for X_2 is positive (negative) so the price pressure on X_2 is up (down) according to the theory. The price space is thus partitioned into four regions in which simple excess-demand driven models of dynamics make definite predictions for the direction of price movements. The implied direction of these movements are shown by the small arrows in Figure 2, with the directed lines presenting a simulated price path based on equation (1) and a given initial position.

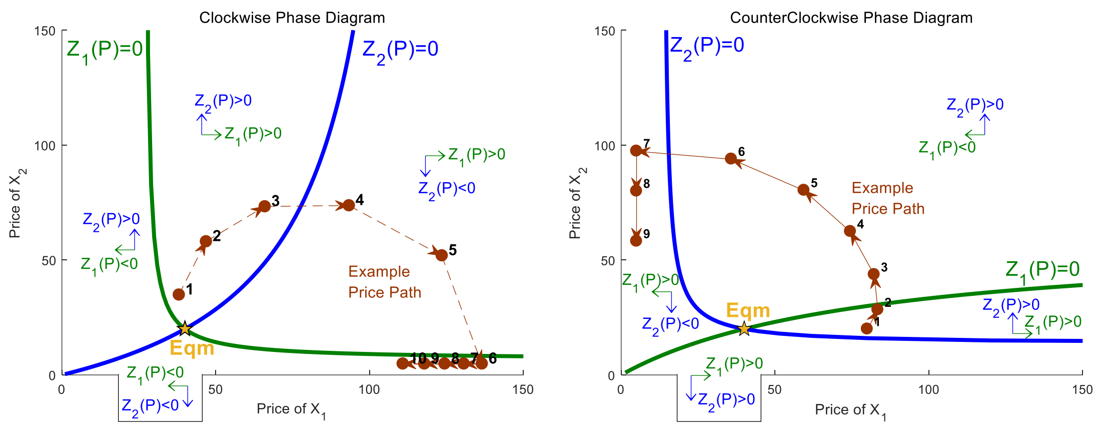

Figure 2: Excess Demand Phase Diagrams for Simple Dynamic Model

From the initial position, the classical model's simple adjustment process predicts dynamics and possibly non-convergent, price dynamics in both treatments. Contrasting Figure 2 for the Clockwise and Counterclockwise specifications identifies the difference in predicted price dynamics for the two treatments. In the Clockwise treatment, classical dynamics suggest prices move in a clockwise manner around the competitive equilibrium, so the angle of prices relative to equilibrium (P_1, P_2) plane declines as prices adjust (until it jumps upon passing zero). In contrast, the same model predicts prices in the Counterclockwise treatment move in the opposite direction, counterclockwise around equilibrium prices.[12]

Note that the example price paths depicted in Figure 2 only represent *expected* changes in prices. The actual price paths will be affected by substantial unmodeled variability, including behavioral artifacts, microstructure noise, and misspecification, leading to transactions executing at a wide variety of prices. The substantial unpredictable component of price dynamics is to be expected in light of market forces limiting potential arbitrage opportunities. Separating this signal from the noise requires econometric evaluation to resolve the empirical question of whether predictions from excess demand models effectively characterize expected price dynamics.

[12] In these figures, as in experimental sessions, we impose a floor on both P_1 and P_2 equal to 5 units of X_3 to avoid technical issues from trades at zero-prices. Also, while certain regions of these diagrams suggest explosive price dynamics, we note that the limited supply of X_3 imposes an effective ceiling on P_1 and P_2.

4. Procedures and Experimental Design

Six separate experiments were conducted, all at the California Institute of Technology in the Laboratory for Experimental Economics and Political Science (EEPS) between November 2002 and July 2003. Each experiment consisted of a number of subjects modulo 3, as we require that there be an equal number of subjects of each type. The actual number of subjects in the experiments ranged from 9 to 18, with Table 3 summarizing the sessions conducted using each treatment specification. Participants included Caltech undergraduate and graduate students, as well students from Pasadena City College, many of whom were familiar with the software from previous (unrelated) experiments, but who did not necessarily have any training in economics.

Types were assigned sequentially to subjects as they logged into the software, and the order in which this occurred was essentially random. Subject payments averaged about $40.00 per subject per experiment. Experiments lasted no more than three hours. Upon arrival in the laboratory, subjects were given written instructions; including both a numeric table and a graphical display of indifference curves that represented their own (and only their own) endowments and induced preferences. In addition, we included an unrelated payoff table that illustrated how to read their true payoff table (which differed across subjects).

Each experiment began with a practice trading period serving several purposes. It acquainted subjects with the computers and software, so that they were comfortable with how to execute bid and ask offers before the paid portion of the experiment began. It also allowed time for the subjects to study their payoff information. Finally, it worked as a device for influencing initial conditions as we requested all trades in the practice period take place at a price of 25 units of X_3,[13] essentially providing a focal point for prices ahead of the first actual period. Thus, the initial starting point would be (25,25).

Following the practice period, each experiment consisted of a number of trading periods, ranging from 9 to 19 periods per session. Each period, in turn, lasted between 8 and 18 minutes. To avoid any "last period" effects, the final period was not announced as such until *after* it had concluded. After the close of the final period, earnings in francs were tallied and converted to dollars via a conversion factor. Subjects were then either paid in cash before they left the laboratory, or else checks were mailed to them shortly thereafter.

[13] We did not establish this focal point for prices in the first session, experiment 021127. We note that these starting prices differ from the initial points in Figure 2, which were chosen specifically to illustrate the cyclical features of the model. We discuss the manifestation of these cyclical dynamics from the training prices, including the role of integer constraints and unmodeled variability in prices, in detail in Appendix A.2.

Table 3: Experimental Sessions

Treatment	Date	Periods	N	Experienced Included
I. Clockwise	11/27/2002	10	18	No
	12/11/2002	14	12	No
	7/17/2003	11	18	Yes
II. Counterclockwise	1/30/2003	12	15	Yes
	4/28/2003	9	15	Yes
	6/20/2003	19	9	Yes

5. Experimental Price Dynamics

We begin our analysis of market outcomes by presenting the price processes observed in the different experimental markets. Figure 3 presents the time-series of the commodities X_1 and X_2 market prices for each executed transaction along with the period-average prices from the 021211 Clockwise and 030428 Counterclockwise treatment sessions.[14] The figure demonstrates clear variability of prices both for each individual transaction, as well as for period average prices, and suggests a cyclical tendency in the relative prices of the two commodities.

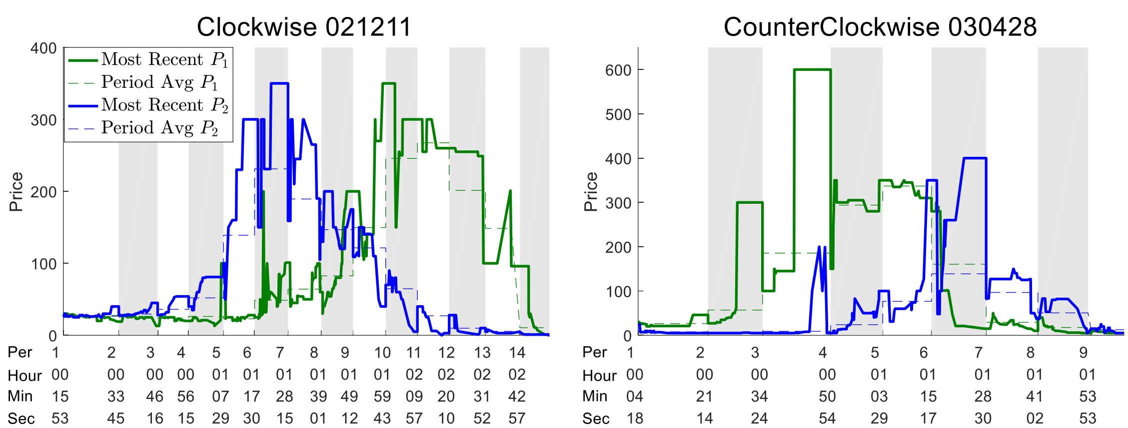

Figure 3: Transaction Price Time Series

A. Period Price Dynamics Relative to Excess Demand Phase Diagrams

We now evaluate price dynamics in the context of the implied phase diagram constructed from excess demand in Figure 2. Figure 4 plots average period prices in the (P_1, P_2) plane, in relation to the phase diagram and predicted dynamics of the classical model. In addition to period-average prices for the 021127 Clockwise and 030130

---

[14] These two sessions illustrate well our main experimental findings. For the sake of parsimony, the text will present several figures using these only two examples to illustrate the observed dynamics of trading and allocations. For completeness, figures from all sessions are presented in Appendices B & C.

Counterclockwise sessions, which discretely smooth out much of the variability from individual transaction prices, Figure 4 plots an exponentially weighted moving average of prices, a smoothed presentation characterizing the instantaneous variability in prices for individual trades. While the movement does not appear to be toward the equilibrium in either treatment, the general pattern appears consistent with classical predictions.

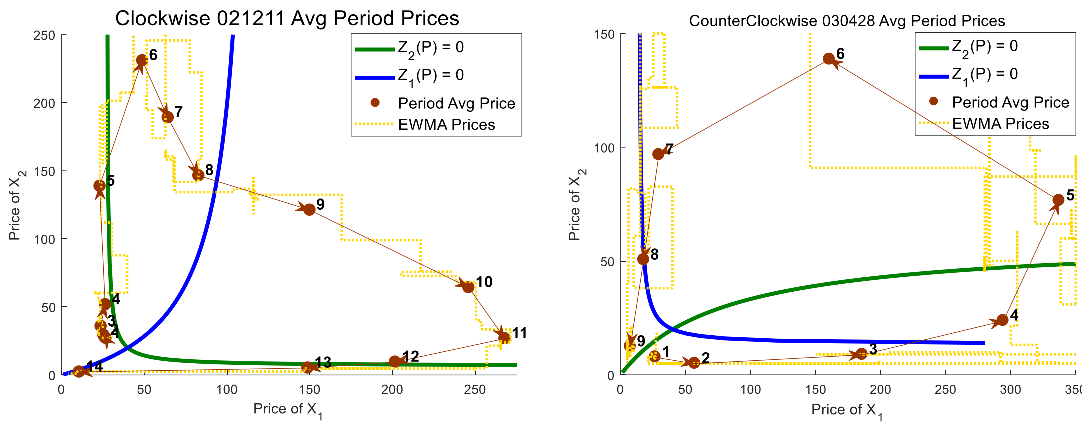

Figure 4: Period Average Prices and Phase Diagram

Figure 5 plots the period average prices for all sessions with the Clockwise and Counterclockwise treatments in the partitioned phase diagram. These paths present the clear impression that prices move in the general direction predicted by theory and provide a convenient illustration of our major results. First, prices in continuous double auctions need not converge to an interior equilibrium. Second, disequilibrium price movements are reasonably well-predicted by measures of excess demand.

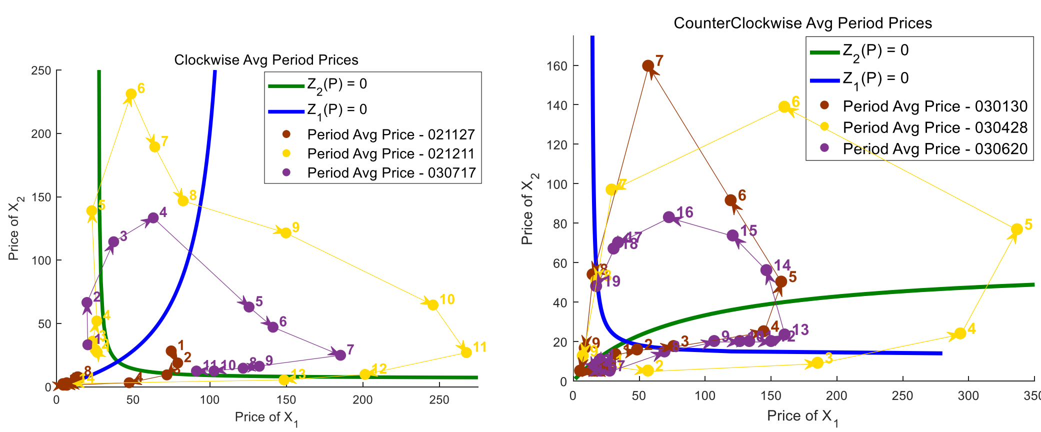

Figure 5. Average Period Prices All Experiments

A detailed evaluation of predicted price movements by excess demand reveals some empirical limitations, as suggested by visual inspection of the Counterclockwise 030428 session period-average prices. At the beginning, price movement proceeds downward and to the right as predicted, but in period three the price of X_2 moves upward slightly, pulling prices across the partial equilibrium line for X_1 and causing a jump in phase. Continuing to follow the progression of this series, notice that the price of X_2 declines between periods 6 and 7 when the model suggests that it should continue increasing.

Many subtle patterns exist in these data, some of which require a generalized classical model for consistent interpretation. The empirical challenge, then, is to identify those patterns that represent predictable price movements from the noise inherent to market mechanisms involving real-time trading. The next sections demonstrate these results formally, statistically evaluating the degree to which equilibrium prices and excess demand predict price movements.

B. Dynamic Inventories, Excess Demand, and Equilibrium

Disequilibrium trades are an important feature of the experimental markets, causing agents' inventories to shift away from their endowments after each transaction. In addition to prices, these dynamic inventories also affect excess demand, introducing a dynamic equilibrium to the experimental setting. To characterize how shifting endowments and changing prices interact to determine excess demand, we introduce the Instantaneous Excess Demand measure. Agent i's Instantaneous Demand at time t is the integer-constrained bundle of commodities that maximizes the agent's utility assuming perfect liquidity at prices $P_{1,t}$ and $P_{2,t}$ when endowed with the agents' contemporary holdings $X_{1,i,t}$, $X_{2,i,t}$, and $X_{3,i,t}$. The Instantaneous Excess Demand for a commodity is then the aggregated demand for that commodity minus its economy-wide endowment.

To illustrate the dynamics of excess demand, Figure 6 plots the time series of instantaneous excess demands for the experiments after accounting for both the evolving inventories and prevailing prices, normalized so as to be expressed as a percentage of the economy-wide endowment of commodities X_1 and X_2.

The excess demand dynamics highlight two important features of the observed transactions. First, we note that the clockwise and counterclockwise treatments demonstrate opposing tendencies in the relative excess demand for commodities X_1 and X_2. Second, observe that the excess demand at the end of each period do not systematically converge to zero indicating trading within a period failed to realize an equilibrium allocation at terminal prices. In addition to the empirical phase diagram from Figure 6, this result suggests that observed trading behavior did not realize an equilibrium allocation.

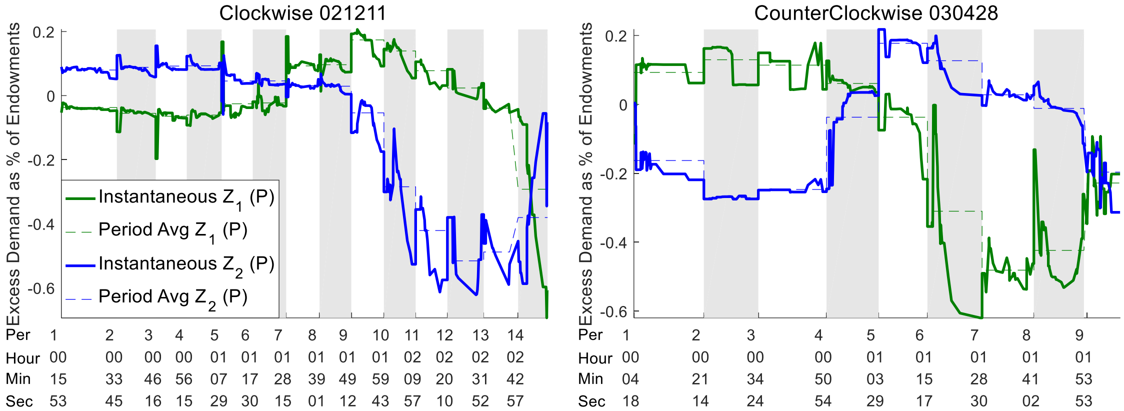

Figure 6: Excess Demand Dynamics

6. Divergence, Allocation Efficiency, and Trading Volume

In this section, we evaluate the extent to which trading in the continuous double-auction mechanism leads to a divergence in prices from the interior equilibrium, as well as the relationship between allocation efficiency, disequilibrium pricing, and trading volume. While these issues are broad ranging with a long history initiated by debates between Edgeworth and Walras, as highlighted by discussion in Walker (1987) and Donzelli (2009), our focus is narrow. We demonstrate first that prices can systematically move away from an interior theoretical equilibrium, a feature reflecting the views of Edgeworth. Specifically, the Euclidian distance between transaction prices and equilibrium prices demonstrates a positive time trend both across periods and for trades occurring within each period. Second, we find that the market allocations' efficiency is high for complex markets, but that some gains from trade persist at the end of each trading period throughout the experiment. It's important to note that this result is not driven by design decisions restricting the length of each trading period, but rather obtains after the effective cessation of trading in any given period.

A. Price Divergence from Equilibrium

Our analysis begins by considering the Euclidean distance between a pair of transaction prices and the interior theoretical equilibrium price of $P^* = (40,\ 20)$.[15] Figure 7 presents the time series of these distances for each of the six sessions, demonstrating that prices can move very far away from equilibrium in the course of trading.

[15] Because transactions occur asynchronously, we interpolate prices between trades simply using the last price at which the commodity sold.

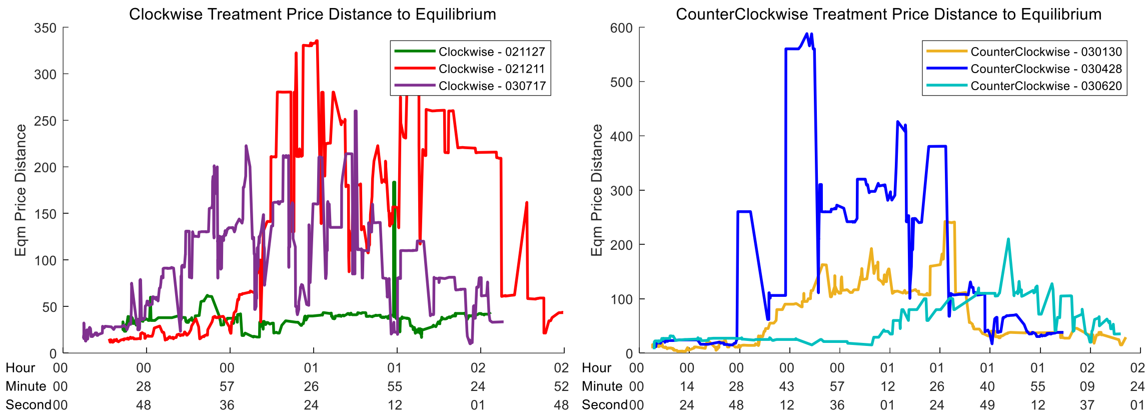

Figure 7: Distance from equilibrium for each session and Treatment

To investigate this property statistically, we estimate the time trend for theoretical equilibrium distance and test the null hypothesis that this trend is weakly negative in Table 4. Panel A presents the results from pooling all sessions of the experiment, with a significant positive trend demonstrating the tendency of transaction prices to move away from the equilibrium. Regression results from individual sessions presented in Panel B largely agree with this tendency, though the smaller sample sizes in each session prevent tests from achieving the statistical significance of the pooled sample. The weakest demonstrated trend appears in Session 021127, which was the sole session where training period prices were not fixed at (25, 25) and prices failed to move away from the origin.[16]

Table 4: The Time Trend of Equilibrium Price Distance

Panel A: Pooled Regression Results

	Intercept	Trend
Coefficient	24.81	1.10
Std Error	8.03	0.25
p-value	<0.01	<0.01

Panel B: Individual Session Regression Results

Clockwise Treatments			CounterClockwise Treatments		
021127	Intercept	Trend	*030130*	Intercept	Trend
Coefficient	34.39	0.03	Coefficient	45.08	0.21
Std Error	1.79	0.02	Std Error	29.20	0.34
p-value	<0.01	0.13	p-value	0.12	0.54

[16] To confirm that this result is not driven primarily by across-period variation in prices, we conducted a paired t-test on the beginning and end prices of the market across all sessions. The test asks if prices are closer to the equilibrium prices, (40, 20), in the first transactions executed at the beginning of the period than they are in the last transactions executed at the end of the period. For these prices, the mean distance was 18.01 (in units of X_3) at the beginning of the experiment, and 36.89 at the end (t = 5.59). The test is significant at p<0.01 for a one-tailed test. The result verifies that equilibrium divergence occurs within each trading period as well as over time across periods.

021211	Intercept	Trend
Coefficient	13.79	1.32
Std Error	43.66	0.55
p-value	0.75	0.02

030428	Intercept	Trend
Coefficient	121.99	0.25
Std Error	73.50	0.98
p-value	0.10	0.80

030717	Intercept	Trend
Coefficient	72.61	0.27
Std Error	25.06	0.30
p-value	<0.01	0.37

030620	Intercept	Trend
Coefficient	15.18	0.56
Std Error	8.17	0.17
p-value	0.06	<0.01

We summarize our conclusions from this subsection in the following result:

Result 1: Price Divergence from Equilibrium
Transaction prices do not converge to theoretical interior equilibrium but instead demonstrate a trend that moves away from equilibrium prices as time progresses and is evident over time across periods and within each period.

B. End-of-Period Allocations and Efficiency

As the previous subsection presented evidence that traded prices need not converge to the competitive equilibrium as defined by parameters. That being the case, the question turns to why the trading stopped at the end of the periods. (1) Did a different equilibrium emerge as a result of disequilibrium trades or (2) did trading not stop but the period ended because of arbitrarily imposed time limits?

The literature points to the possible exhaustion of gains from trade as an important variable. If the market operates as efficiency seeking mechanism it would stop when gains no longer exist. The possibility is posed by Mukherji and Guha (2011) and by Mukherji (2012) who establishes the possibility that equilibration can emerge through holdings modifying, disequilibrium exchanges such that a competitive equilibrium exists given the holdings of the moment. Of course trading could have ended because the time allowed for trading ended. Recall, this is a real time market process.

Gains from exchange in the experiment are measured in terms of additions to “take home” money acquired by trading initial endowments for other commodities. In the Scarf environment the initial endowments are worth nothing in terms of the money received in terms of the financial incentives used to induce preferences. Traders are endowed with units of commodity with no value unless complement commodities are also held. Exchange of commodities can increase the amount of money that the subject makes from the experiment. The exchanges produce income or wealth that can be summed and interpreted as net social benefits in a cost-benefit sense.

We evaluate end-of-period allocations using two devices, one that measures net social benefits and another that characterizes Pareto efficiency. Our net social benefits measure simply totals the dollar earnings for all agents and compares that to the total dollar earnings realized at the competitive equilibrium allocation. To consider Pareto efficiency, we first define an agent's "residual holdings" as the units of a commodity held by the agent that provide zero marginal utility to that agent. Of course, the units can be valuable to someone else so, in a sense, these units are a pure waste. We then total these residual holdings across agents and compare them to the total endowment for the market, so that all measures can be expressed in percentage terms.

Figure 8 presents the end-of-period efficiency and residual holdings across all sessions. Averaging across all sessions, the end of period allocations realized approximately 75% of the net social benefits delivered by the equilibrium allocation. While this level is lower than usually found in single market experiments, the level is comparable to the efficiencies of multiple markets. The average residual holdings were approximately the same for each security, at around 25%, though Figure 8 reveals variation in that average across periods and sessions. The efficiency measure illustrates a pattern atypical of multimarket experiments starting at high levels of efficiency that are reduced in subsequent periods. We conjecture this pattern reflects early trading near competitive equilibrium prices and the subsequent divergence as is illustrated in Figure 7.

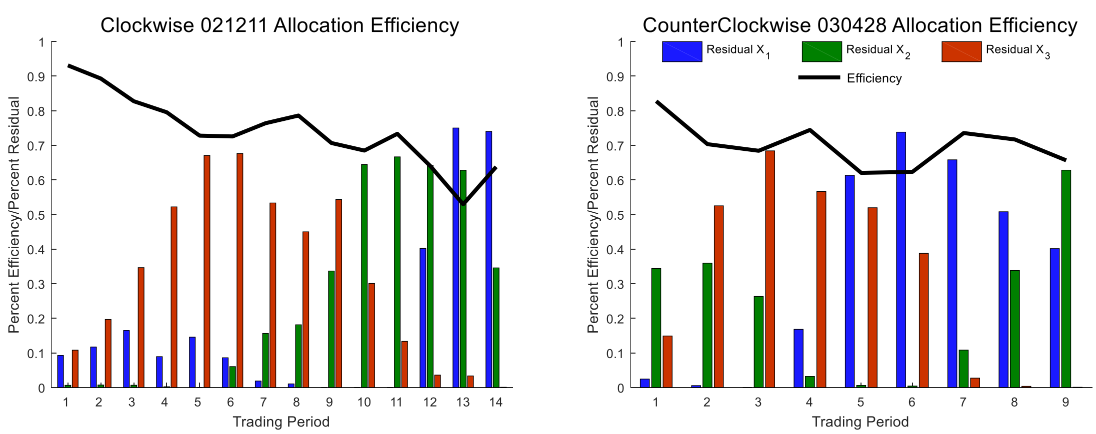

Figure 8: Period-End Allocation Efficiency

While trading exhausts much of the gains from trade, the result suggests that gains from trade exist at the end of each session that are not realized by subjects in the experiment. Trading did not stop because of equilibration due to a complete lack of gains from exchange or that gains from exchange were completely exhausted. This feature of the data is also supported by the data in Figure 6 in Section 4B that demonstrates the existence of instantaneous excess demand and supplies at the end of periods. Strictly speaking, the markets were not at a competitive equilibrium given their holding at the end of the period.

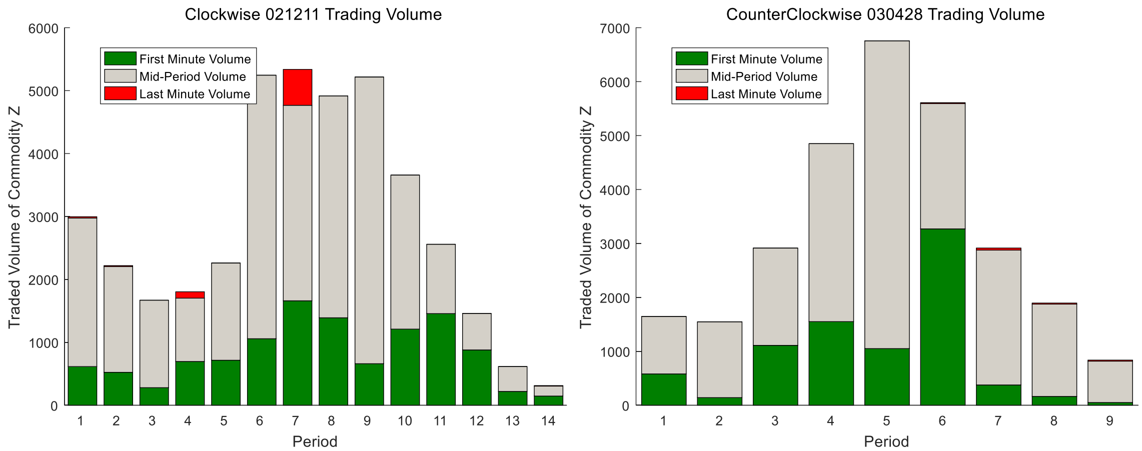

Figure 9: Early-vs-Late Transaction Volume by Period

Trading did not stop due to the arrival of the end of the period and insufficient time to trade. Figure 9 decomposes trading volume for each period into transactions that occur in the first minute, last minute, and the intervening duration of the period. Transactions occur in the last minute of a period in only 35% of the periods from the experiment and those transactions that do occur tend to be small in total value. Importantly, the failure of the markets to realize these gains from trade is not due to a design decision limiting the duration of each trading period, but rather to agents' decisions to cease transacting.

Result 2: Unrealized Gains from Trades Persist
Trading did not stop because gains from exchange were completely exhausted or because no time for trading remained.

A different understanding of why trading stopped is suggested by the complex transactions required to realize those gains. The required transactions are non-trivial, requiring multiple counter-parties. Each agent type realizes utility from only two of the three commodities and, at the end of each period, very few agents hold inventories of the commodities from which they receive zero utility. Across all sessions, only 14.6% (17.3%) of periods ended with an agent of type 1 (type 2) maintaining a dispreferred position in X_1 (X_2) from which they receive no utility regardless of their other holdings. Of those sessions that end with agents holding dispreferred positions, these holdings are predominantly held as shares of the numeraire commodity X_3 by the type 3 agent who receives no utility from it but is unable to purchase shares of (perhaps both) other goods at prevailing prices.

These positions limit the availability of bilateral trades that could enhance net social benefits, implying trilateral transactions would be required to realize allocative efficiency. Given this complexity and need for coordination, trading volume dissipates near the end of the period. This interpretation supports the Mukerji (2012) model of equilibria emerging from the dynamic adjustments. It also suggests the possible importance of general equilibrium adjustments as excess demand order flow in complementary markets create expectations that units will become available and thus create value in units held in a given market.

7. Models of Non-Equilibrium Dynamic Adjustment

Though the previous section presents the result that observed transaction prices deviate substantially from the theoretical equilibrium. We begin this section by introducing two disequilibrium models to organize our empirical analysis.

Our analysis reveals two striking results. First, we find that predicting relative price changes using the generalized classical model best describes the observed price paths. Second, we find that excess demands in one market have very little influence on price dynamics in the other market. This latter finding provides a novel empirical evaluation of the power of partial equilibrium models in a setting were general equilibrium adjustments exist.

A. A Brief Outline of Models of Disequilibrium Price Dynamics

The introductory discussion earlier presented the "Classical Model" of disequilibrium price dynamics in which the rate at which prices change are proportional to their excess demand. In this formulation, recall that the price dynamics from equation (1) can be represented by the difference equation:

$$\begin{bmatrix} \dot{P}_{1,t} \\ \dot{P}_{2,t} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{pmatrix} Z_1\left(P_{t-1}\right) \\ Z_2\left(P_{t-1}\right) \end{pmatrix}.$$

A simple generalization embeds the Classical Model as a special case of the "Generalized Classical Model,"

$$\begin{aligned} \dot{P}_1 &= a_{11} Z_1\left(P_1, P_2\right) + a_{12} Z_2\left(P_1, P_2\right) \\ \dot{P}_2 &= a_{21} Z_2\left(P_1, P_2\right) + a_{22} Z_2\left(P_1, P_2\right), \end{aligned} \tag{2}$$

The Generalized Classical Model allows for price adjustments of markets linked by a "linked adjustments principle." That is, the adjustment in a single market depends on the state of disequilibrium (as measured by excess demand) in other markets. The classical model is the special case that satisfies the restrictions $a_{11} > 0,\ a_{22} > 0,$ and, $a_{12} = a_{21} = 0$.

We originally used the generalization as a technical tool to evaluate the magnitude of deviations from the Classical Model in which the off diagonal elements are zero. However, after thought and data analysis we discovered the generalization play a deeper role in the theory of dynamic equilibration. Introducing possible sensitivity for price adjustments to the degree of disequilibrium in other markets, measured by the size of the excess demands, provides insight into the possible role of uncertainty in disequilibrium dynamics. While supply might be greater than demand for commodity X_1, the disequilibrium in the market for X_2 might attenuate the rate at which P_1 decreases or even cause P_1 to increase rather than decrease. Walras and others tended to reject this as a possibility and postulated the "fundamental principle" that the direction of price change of a given commodity depends only on the sign of its own excess demand.[17]

Excellent reviews of classical price dynamics are presented by McKenzie (2002) and by Mukherji (2002, 2003). The models presented by equations (1) and (2) present special cases of theories that have the following form:

$$\dot{P} = A(P)Z(P)$$

Here, $\dot{P}$ represents the change in prices over time, P is the price vector. We refer to $A(P)$ as an adjustment matrix of coefficients that may depend on prices P, and $Z(P)$ is a vector of excess demands as a function of prices. Though the theory generalizes to any number of commodities, we focus on the two-dimensional price setting implemented here for ease of exposition.

The primary feature of this model is that price changes depend upon P through the adjustment matrix $A(P)$ in addition to the functional relationship by which prices enter the excess demand functions. Though the dependence of the adjustment matrix on prices could take any form, we look the literature to identify plausible restrictions that impose some structure on this relationship.[18] In particular, we explore specifications of the Classical and Generalized Classical models where the elements of $A(P)$ vary with P so as to characterize relative price dynamics. Specifically, consider a "Generalized Relative Model" in which price dynamics take the following form:

$$\begin{pmatrix} \dot{P}_1 \\ \dot{P}_2 \end{pmatrix} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} Z_1(P) \\ Z_2(P) \end{pmatrix} \tag{3}$$

In the Generalized Relative Model, the function $A(P)$ decomposes into a matrix that contains the prices and a matrix of constants which pre-multiplies the excess demand

[17] Walras (1954, p.85) states three suppositions that collectively state that the sign of the excess demand and the sign of price changes will be the same. As mentioned above, Hicks appears willing to postulate the existence of a linkage. As will be mentioned below, Edgeworth presents a different opinion based on a different model of price adjustments.

[18] As we are unaware of any attempt to study this model in its most general form, we focus on classes of special cases, although the literature is rich with discussion about the conditions under which less information is required for convergence. See Mukherji (1995) for a summary of recent literature, and for a treatment of stability in three commodity (two prices) models see Mukherji (2004).

functions. This model of price dynamics can be equivalently expressed so that the percentage change in price depends on excess demands through a constant adjustment matrix. To illustrate in the two commodity case for any single commodity, *i*, the adjustment process in equation (3) can be written as:

$$\frac{\dot{P}_i}{P_i} = a_{i1} Z_1(P) + a_{i2} Z_2(P),\; i = 1, 2.$$

Of course, this model can be further refined by hypotheses focused on the numbers $a_{ij.}$ In particular, the Classical Relative Model presents a restricted case of equation (3) in which the off diagonal elements are restricted to be zero and the diagonal elements positive. Table 6 summarizes this section, consolidating the dynamic models we consider for ease of reference. The previous section illustrates challenges presented by trying to fit the data to the Equilibrium model. The next subsection begins evaluating the absolute models by verifying the relevance of excess-demand driven dynamics in predicting the direction of price changes. Our analysis then turns to estimating the data generating process under the Generalized Absolute and Relative Model to test the Classical and Identity restrictions on the adjustment matrix. Finally, we consider which of the models we study here best describe the data generating process.

Table 6: Predictive Expectations for Price Dynamics

Specification	*Absolute Model*	*Relative Model*
Classical	$\begin{bmatrix} \dot{P}_{1,t} \\ \dot{P}_{2,t} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{pmatrix} Z_1(P_{t-1}) \\ Z_2(P_{t-1}) \end{pmatrix}$	$\begin{bmatrix} \dot{P}_{1,t} / P_{1,t-1} \\ \dot{P}_{2,t} / P_{2,t-1} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{pmatrix} Z_1(P_{t-1}) \\ Z_2(P_{t-1}) \end{pmatrix}$
Generalized	$\begin{bmatrix} \dot{P}_{1,t} \\ \dot{P}_{2,t} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} Z_1(P_{t-1}) \\ Z_2(P_{t-1}) \end{pmatrix}$	$\begin{bmatrix} \dot{P}_{1,t} / P_{1,t-1} \\ \dot{P}_{2,t} / P_{2,t-1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} Z_1(P_{t-1}) \\ Z_2(P_{t-1}) \end{pmatrix}$

B. Excess Demand Dynamics Predict Price Movement Direction

Our analysis in the previous section suggests that prices from the double auction mechanism diverge from theoretical equilibrium prices. We now present evidence that models based on excess demand accurately predict the direction of price movements, as suggested by the phase diagrams in Figures 4 and 5. We consider here two nonparametric tests to verify the link: a "clockhand" test that applies to individual transaction data and a "sign" test analyzing period-level price dynamics.

The "clockhand" test simply recenters prices so that equilibrium lies at the origin and measures the angle in price space between where the data started relative to the equilibrium and where the prices are at any instant of time. A line segment connecting current prices to the equilibrium functions as the hand of a clock, and as prices change, that line segment rotates around the equilibrium. Anderson, et. al., (2004) presents a geometric interpretation of this analysis wherein the clockhand test measures the accumulated rotation of prices over time. As a non-parametric test robust to both boundary restrictions and asynchronous trades, the clockhand test can incorporate the entire time-series of individual transactions from all periods and sessions.

Figure 10 shows the cumulative angle changes based on individual transactions in all 6 sessions. There is a clear separation between the clockwise and counterclockwise treatments. In addition, note that 2 of the counterclockwise treatments resulted in cumulative angle changes greater than 2π. I.e., in two of the sessions, the price orbit completed one cycle.

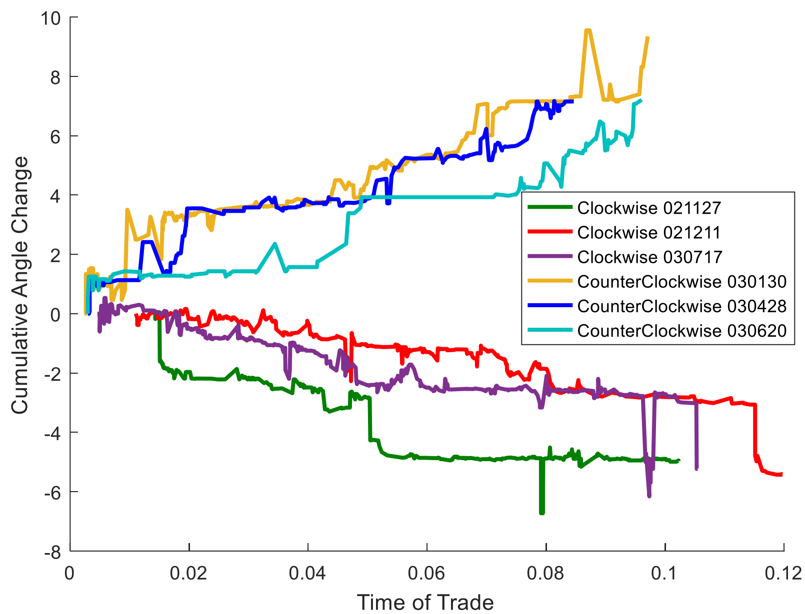

Figure 10: Clockhand Model Plotting Cumulative Angle of Price Changes

The sign test is a simple binomial test that counts the instances where the sign of the price change in a given trade matches the sign of the excess demand in both markets, i.e., whether $\text{sign}(P_{1,t} - P_{1,t-1}, P_{2,t} - P_{2,t-1}) = \text{sign}(Z_1(P_{t-1}), Z_2(P_{t-1}))$. Under the null hypothesis that price dynamics are not predictable by excess demand, this event has a 0.25 probability of occurring. This test is sensitive to both boundary effects *and* asynchronous trades and, therefore applies to individual transactions as well as period average prices. Pooling over all sessions, there were a total of 124 data points, of which in 52 trials the price change was predicted correctly by excess demand in both the X_1 and X_2 markets. Compared to a random prediction expecting 31 correct predictions, the test was significant at a p = 4.11e-5.

Result 3: Excess Demand Dynamics Predict Price Movements
Prices tend to move in the direction predicted by excess demand, both at the individual transaction level and across periods over time.

C. Accommodation and Linkage in the Absolute Model

We now evaluate the predictive power of the Generalized Absolute Model to characterize price dynamics. While excess demands predict well the direction of price movement, we now test whether they also effectively predict the magnitude of observed price changes. Generalized Absolute Model applies to the experimental data taking difference equation (5) with the instantaneous excess demand function as the conditional expectation for transactional price changes in structural estimation equations:

$$\begin{pmatrix} P_{1,t} - P_{1,t-1} \\ P_{2,t} - P_{2,t-1} \end{pmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} Z_{1,t-1}\left(P_{t-1}\right) \\ Z_{2,t-1}\left(P_{t-1}\right) \end{pmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}. \tag{7}$$

The noise term $\varepsilon_t \equiv \left[\varepsilon_{1,t}, \varepsilon_{2,t}\right]'$ is assumed to be mean zero and uncorrelated with past information (order flow, transaction volume, and prices), satisfying the usual conditions for regression analysis.

We present results from estimating the econometric models equation-by-equation allowing intercepts to vary across sessions, using FGLS to account for Autoregressive Conditional Heteroscedasticity within each session. The regressand corresponds to price changes at the transaction level, winsorized absolute price changes for X_1 and X_2 at 50 and 25 units of X_3, respectively, to control the influence of outliers. The regressors consist of excess demand measured instantaneously based on the last available market prices and inventories.[19]

Table 7: Estimated Absolute Model Excess Demand Coefficients and Significance

	Coefficient	Standard Error	t-Statistic	p-Value
a_{11}	5.41 E-02	1.63 E-02	3.32	<0.01
a_{12}	1.32 E-02	8.97 E-03	1.47	0.14
a_{21}	6.07 E-03	1.00 E-02	0.60	0.55
a_{22}	1.47 E-02	4.71 E-03	3.11	<0.01

[19] We evaluated several other specifications, including OLS, Seemingly Unrelated Regressions, and fixed effects, as well as observational weighting (quantity weighted and time-weighted), with qualitatively similar results that differ mainly in coefficients' estimated standard errors. Introducing session-level fixed effects for coefficients results in noisy estimates as the specification fails to take advantage of the information available from the different excess demand dynamics in the clockwise and counterclockwise treatments. For all tables using pooled results, we present session-level results in Appendix C.

Table 7 presents the estimated coefficients and associated significance measures for the Instantaneous Excess Demand of X_1 and X_2, with three key findings regarding the Classical Model. First, Instantaneous Excess Demand for both goods are significant drivers of own-price changes, i.e., the coefficients a_{11} and a_{22} are both statistically significant and positive. Second, neither of the off-diagonal coefficients, a_{12} and a_{21}, are significant, suggesting excess demand conditions in one market has a negligible effect on price dynamics in the other. Calculating an F-Test for the joint restriction, $a_{12} = 0 = a_{21}$ in the SUR specification is only weakly rejected at the 10% level with a p-Value of 0.08.

Combined, these observations suggest supporting evidence for the Classical Restrictions in the Generalized Classical Model. In particular price changes reflect own excess demand and not the excess demand in other markets as postulated by partial equilibrium theories. The next result summarizes the findings of this subsection.

Result 4: Absolute Model Estimates Support Classical Restrictions
The estimated coefficients in the Generalized Absolute Model are consistent with the Classical Model's restrictions:

- Excess demand for a good has a significant impact on expected price changes for that good, supporting price adjustment models driven by partial equilibrium dynamics.
- Cross-excess demand coefficients in the adjustment matrix are much smaller than own-excess demand coefficients and the hypothesis restricting these coefficients to be zero is not rejected at conventional significance levels.
- Walras' Fundamental Principle that the expected sign of a commodity's price change aligns with the sign of its excess demand is violated only in states of extreme disequilibrium and satisfied in 69% of the sample observations.

D. Accommodation and Linkage in the Relative Model

Though the regressand in the Relative Model as presented in Table 7 differs from that of the Absolute Model, we can reformulate the Relative Model's structural equations to enforce consistency:

$$\begin{aligned} P_{1,t} - P_{1,t-1} &= a_{10} P_{1,t-1} + a_{11} P_{1,t-1} Z_{1,t-1}\left(P_{t-1}\right) + a_{12} P_{1,t-1} Z_{2,t-1}\left(P_{t-1}\right) + \varepsilon_{1,t} \\ P_{2,t} - P_{2,t-1} &= a_{20} P_{2,t-1} + a_{21} P_{2,t-1} Z_{1,t-1}\left(P_{t-1}\right) + a_{22} P_{2,t-1} Z_{2,t-1}\left(P_{t-1}\right) + \varepsilon_{2,t} \end{aligned} \qquad (8)$$

Under this specification, we ensure that the regressands in all of our estimation equations are consistent with one another (with the small expense of accommodating heteroscedasticity), a consistency that will prove useful in the next section comparing different model specifications.

From the structural regression equation (8), we can apply the same estimation strategy adopted in the previous section to the Relative Model. The estimated parameters appearing in Table 8 demonstrate a similar relationship to the Relative Model's results in Table 7. The adjustment matrix coefficients for own excess demand (a_{11} and a_{22}) are much larger than those on cross-excess demand (a_{12} and a_{21}), presenting material support for partial equilibrium adjustment dynamics.

Table 8: Estimated Relative Model Excess Demand Coefficients and Significance

	Coefficient	Std Error	t-Statistic	p-Value
a_{11}	2.41 E-03	3.60 E-04	6.71	<0.01
a_{12}	4.60 E-04	1.08 E-04	4.27	<0.01
a_{21}	6.12 E-04	1.90 E-04	3.23	<0.01
a_{22}	1.42 E-03	1.75 E-04	8.12	<0.01

In this specification, these cross-excess demand coefficients are estimated with sufficient precision to statistically reject the dominant diagonal restriction is rejected statistically. While the partial equilibrium model receives support, the general equilibrium adjustments can be detected in this specification. As an empirical phenomenon, these adjustments could arise from behavioral artifacts that aren't included in the abstract model, notably in how expectations of future liquidity could be informed by excess demand in other markets.

Result 5: Relative Model Estimates Statistically Reject Classical Restrictions
The estimated coefficients in the Generalized Relative Model statistically deviate from Classical Model restrictions while supporting Walras' Fundamental Principle:

- Excess demand for a good has a significant impact on expected price changes for that good, supporting price adjustment models driven by partial equilibrium dynamics.
- Cross-excess demand coefficients in the adjustment matrix are much smaller than own-excess demand coefficients though the hypothesis restricting these coefficients to be zero is rejected at conventional significance levels.
- Walras' Fundamental Principle that the expected sign of a commodity's price change aligns with the sign of its excess demand is violated only in states of extreme disequilibrium and satisfied in 86% of the sample observations.

The estimated coefficients suggest that partial equilibrium influences will dominate general equilibrium influences so long as disequilibrium does not generate severe imbalances in excess demands. In the Generalized Relative Model, Walras' Fundamental Principle that the sign of a good's expected price changes matches the sign of its excess demand holds for prevailing prices and inventories during 86% of observed transactions.

8. Conclusion: Interpretations and Implications

This paper explores market price dynamics under challenging conditions of an unstable equilibrium in which traditional models of price equilibration fail to converge to a unique interior equilibrium. However, the traditional models provide predictions of dynamics that allow the experimental study of underlying principles of dynamics. We discover similar non-convergence in experimental markets, with transaction prices moving away from equilibrium and gains from trade persisting throughout the experiment session. Further, we find that frictionless models of price equilibration provide a useful predictive model of price dynamics. These results underscore the positive value of equilibration dynamics for economic analysis in multiple market settings even in settings that do not satisfy all assumptions underlying the equilibration model.

In estimating models of price dynamics, we are able to test the sensitivity of prices to disequilibrium in outside markets. In so doing, we are able to quantify the importance of partial equilibrium adjustments on prices from shocks to excess demand for that good relative to general equilibrium adjustments on prices from shocks to excess demand for other goods. Though statistical estimates reject the absence of general equilibrium adjustments, their estimated magnitudes are small compared to the first-order partial equilibrium adjustments. As a test of Walras' Fundamental Principle, we find the expected sign of a commodity's price change accords with the sign of its excess demand in 83% of the sample.

These results have implications that extend beyond testing theories of disequilibrium price dynamics. In contrast to previous results, we demonstrate that prices in the commonly adopted continuous double auction with multiple markets need not converge and may not realize efficient allocations on the order of magnitudes often found in experiments with single markets and leads to questions about the role of coordination as part of multiple market equilibration. Our findings also suggest that dynamic theories of equilibration can be useful in identifying the conditions under which these phenomena may occur.

In applied settings ranging from financial markets to industrial organization, economic analysis often explicitly or implicitly relies on ignoring general equilibrium adjustments on isolated markets. It is difficult to conceive how researchers could practically account for the phenomena in the field. However, experimental markets provide a viable setting for exploring the relative magnitude of partial and general equilibrium adjustments in economic analysis. For now, the experiments provide data verifying Walras' Fundamental Principle under the demanding conditions and invite exploration to other settings.

REFERENCES

Anderson, Christopher M., C. R. Plott, and K. I. Shimomura, and S. Granat, "Global Instability in Experimental General Equilibrium: The Scarf Example." *Journal of Economic Theory*, 115(2), (April 2004):209-249.

Donzelli, Franco, "Edgeworth vs. Walras on Equilibrium and Disequilibrium," *UNIMI - Research Papers in Economics, Business, and Statistics*, unimi-1096, Universitá degli Studi di Milano, (2009).

Gintis, Herbert, "The Dynamics of General Equilibrium." *The Economic Journal* 117(523), (2007)

Hirota, Masayoshi, "On the Stability of Competitive Equilibrium and the Patterns of Initial Holdings: An Example." *International Economic Review* 22, (1981):461-7.

McFadden, Daniel, "On Hicksian Stability", *Value, Capital and Growth*, J.N. Wolfe (ed.), 329-351, University Press: Edinburgh, 1969.

McKenzie, Lionel W., *Classical General Equilibrium Theory*, MIT Press, Cambridge, Mass. 2002.

Mukherji, Anjan, "A Locally Stable Adjustment Process," *Econometrica* 63(2), (March, 1995):441-448.

Mukherji, Anjan, *An Introduction to General Equilibrium Analysis*, Oxford University Press, India, 2002.

Mukherji, Anjan, Competitive Equilibria: Convergence, Cycles or Chaos, Centre for Economic Studies and Planning, Jawaharlal Nehru University, New Delhi, PRELIMINARY (2003).

Mukherji, Anjan, "A Global Stability Condition: Generalized Law of Demand", presented at: International Conference on Economic Theory, Tokyo, March 2004, organized by Keio University and Kyoto University.

Mukherji, Anjan, "The Second Fundamental Theorem of Positive Economics", *International Journal of Economic Theory*, 8, 2012, 125-138.

Mukherji, Anjan, “General Equilibrium: A Status Report”, Conference on History of Economics held at Azim Premji University, Bengaluru. November 17-19, 2016.

Mukherji, A., and S. Guha, (2011), *Mathematical Methods and Economic Theory*, Oxford University Press, New Delhi.

Plott, Charles R., “Market Stability: Backward Bending Supply in a Laboratory Market.” *Economic Inquiry* 38(1), (2000):1-18.

Plott, Charles R., “Equilibrium, Equilibration, Information and Multiple Markets: From Basic Science to Institutional Design,” Nobel Symposium Behavioral and Experimental Economics, December 2001.

Plott, Charles R., and G. George, “Marshallian vs. Walrasian Stability in an Experimental Market.: *Economic Journal* (May 1992):437-60.

Plott, Charles R., and P. Gray, “The Multiple Unit Double Auction.” *Journal of Economic Behavior and Organization* 13, (1990):245-58.

Plott, Charles R., and Kirill Pogorelskiy. "Call Market Experiments: Efficiency and Price Discovery through Multiple Calls and Emergent Newton Adjustments." *American Economic Journal: Microeconomics*, 9(4), (2017): 1-41.

Plott, Charles R., and J. Smith, “Instability of Equilibria in Experimental Markets: Upward-Sloping Demands, Externalities, and Fad-Like Incentives.” *Southern Economic Journal* 65(3), (January 1999):405-426.

Plott, Charles and Vernon Smith. “An Experimental Examination of Two Exchange Institutions.” *Review of Economic Studies*, 45(1), (1978): 133-153, https://EconPapers.repec.org/RePEc:oup:restud:v:45:y:1978:i:1:p:133-153..

Plott, Charles R., and Vernon L. Smith (editors), *Handbook of Experimental Economics Results, Vol 1*, North-Holland, 2008.

Scarf, Herbert, “Some Examples of Global Instability of the Competitive Equilibrium. *International Economic Review* 1, (1960):157-71.

Smith, Vernon L., “Experimental Auction Markets and the Walrasian Hypothesis” *Journal of Political Economy*, (1965).

Walker, Donald A. "Edgeworth versus Walras on the Theory of Tatonnement." *Eastern Economic Journal*, 13(2), (1987):155–165.

Walras, Leon. *Elements of Pure Economics* (William Jaffé, Trans.), 1954. London: Routledge (Original work published 1899)

APPENDIX A: Theoretical Background and Experimental Details

A.1. Notes on Experimental Design and Parameters

The parameters chosen for the experiments reflected considerable research on the various possibilities. This appendix provides an overview that attempts to help the reader understand the parameters used and provides those interested with suggestions about additional experiments and tests.

Four parameters are used to form preferences and initial endowments across the experimental series. These parameters $\{\alpha, \beta, \gamma, q\}$ interacted with preferences and endowments. The interactions with preferences are as follows. Notice that α is a scaling parameter for X_2, β is a scaling parameter for X_3 and γ is a scaling parameter for X_1. The parameter q operates on individuals to change the value of different goods across the individuals. The functions studied when in parametric form are:

$$U_1 (X_2, X_3) = \min [X_2/q\alpha, X_3/\beta]$$
$$U_2 (X_1, X_3) = \min [X_1/\gamma, X_3/q\beta]$$
$$U_3 (X_1, X_2) = \min [X_1/q\gamma, X_2/\alpha]$$

The choice of experimental design also involves an interaction of the four parameters with initial endowments. The following example illustrates the material that will be presented in the table in the next section. The example is for the case of clockwise unstable parameters that were actually used in the experiments.

Clockwise Parameters: $(\gamma, \alpha, \beta, q) = (20, 40, 800, 1/3)$

	Type 1	Type 2	Type 3
Preferences:	$\min\{3X_2/40, X_3/800\}$	$\min\{X_1/20, 3X_3/800\}$	$\min\{3X_1/20, X_2/40\}$
Endowments:	$E_1=(0,0,\beta)=(0,0,800)$	$E_2=(\gamma,0,0)=(20,0,0)$	$E_3=(0,\alpha,0)=(0,40,0)$

The predictions for this set of parameters are:

(i) Equilibrium: $(P_1, P_2) = (\beta/\gamma, \beta/\alpha) = (40, 20)$

(ii) Dynamics: Unstable time path moving in a clockwise direction

Table A1 provides a pattern of parameters that created a background for the specific choice of parameters for implementation. Parameters that theoretically lead to closed cycles and to stable paths have been studied by Anderson, et. al., (2003) and by Plott (2001). While existing studies did not use the parameters in the table, the parameters used in those studies did lead to the same qualitative implications for system behavior as the parameters in the table. Thus, we make no attempt here to study parameters that theoretically lead to stability or theoretically lead to closed cycles. The question posed here is whether or not divergence can be observed in practice so the focus was on parameters that theoretically lead to divergence. Those parameters correspond to specifications in the upper left and lower right of Table A1, below. As can be seen, the difference between the clockwise and counter-clockwise treatments resides in the choice of *q* and the choice of initial endowments.

Table A.1.1: General Parameter Set for Stability Analysis

Q	Endowments $(\gamma, \alpha, \beta) = (20,40,800)$	
	type one $(\mathbf{x_1,x_2,x_3}) = (0, \alpha, 0)$ type two $(\mathbf{x_1,x_2,x_3}) = (0, 0, \beta)$ type three $(\mathbf{x_1,x_2,x_3}) = (\gamma, 0, 0)$	type one $(\mathbf{x_1,x_2,x_3}) = (0, 0,\beta)$ type two $(\mathbf{x_1,x_2,x_3}) = (\gamma, 0, 0)$ type three $(\mathbf{x_1,x_2,x_3}) = (0,\alpha,0)$
q>1 q = 3 for experiments	unstable counterclockwise equilibrium prices (40,20)	Stable equilibrium prices (40,20)
q=1	limit cycle counterclockwise equilibrium prices (40,20)	limit cycle clockwise equilibrium prices (40,20)
q<1 q = 1/3 for experiments	Stable equilibrium prices (40,20)	unstable clockwise equilibrium prices (40,20)

Table A2 contains the parameter set for the experiments conducted. The information in this table is essentially the same as the information in Table 1 in the text. It is included here for the convenience of readers who want to compare the parameters that were implemented to the more general possibilities.

Table A.1.2: Preferences And Endowments

	Type i = 1,2,3	$U^i(x_i,y_i,z_i)$	endowments $\omega_1 = (\mathbf{x_i,y_i,z_i})$	Remarks
Clockwise: q=1/3, $(\gamma,\alpha, \beta) = (20,40,800)$; Equilibrium $P_x = \beta/\gamma$, $Py = \beta/\alpha$				
	1	min{3y/40,z/800}	ω_1=(0,0,800)	The classical model predicts divergence with tendencies in a clockwise direction.
	2	min{x/20,3z/800}	ω_2=(20,0,0)	
	3	min{3x/20,y/40}	ω_3=(0,40,0)	
Counterclockwise: q = 3, $(\gamma,\alpha, \beta) = (20,40,800)$				
	1	min{y/120,z/800}	ω_1=(0,40,0)	The classical model predicts divergence with tendencies in a counterclockwise direction.
	2	min{x/20,z/2400}	ω_2=(0,0,800)	
	3	min{x/60,y/40}	ω_3=(20,0,0)	

A.2: Initial Conditions and Cyclical versus Explosive Behavior

The cyclical price patterns depicted in Figure 2 depend in part on the initial conditions, which in the current market implementation could instead give rise to explosive price patterns. Excess demand dynamics in a continuous market without noise presented in Figure A.2.1 demonstrates the potential for explosive, rather than cyclical price dynamics depending on where prices initiate. Notably, from the training price conditions (25, 25), the clockwise model predicts such an explosive dynamic. Further, transactions occur at prices throughout the price space over the course of the entire experiment, a result that's incompatible with the precise predictions of the difference equations.

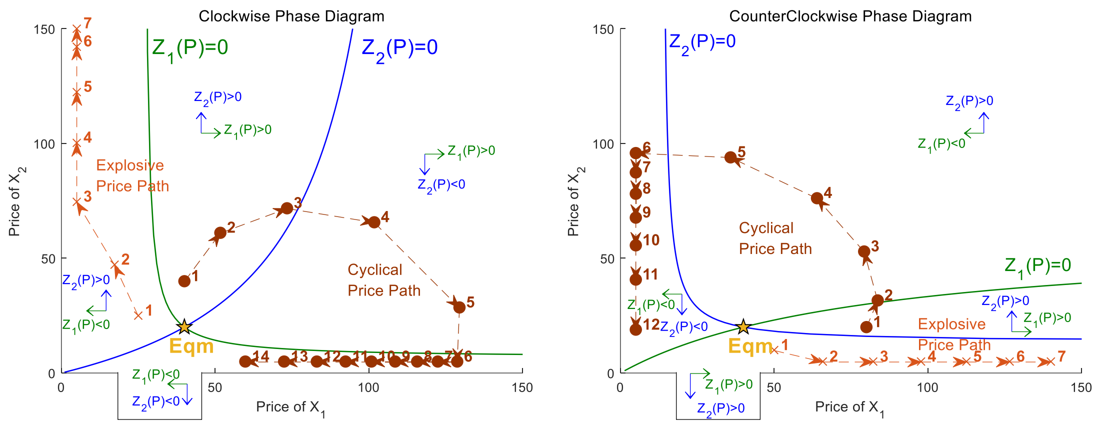

Figure A.2.1: Cyclical and Explosive Price Paths

In practice, three features of the markets implemented in the experiment can give rise to phase changes that induce cyclical price dynamics even when prices may lie in the explosive region. First, the limited number of units of X_3 in the economy function as a price ceiling for P_1 and P_2 that puts a ceiling on the degree to which explosive prices can be observed. Second, unmodeled variation in the prices at which trades execute is of sufficient magnitude to "jump phase" and move prices into a region of the phase diagram in which cyclical dynamics dominate. Third, constraining trades to integer units of X_1 and X_2 complicates the excess demand dynamics allowing for cyclical behavior to be observed from a larger set of starting conditions and substantially expanding the set of equilibrium prices.

A.2.1 Effective Price Ceilings Bound Explosive Tendencies

A simple practical feature of the markets we study prevent us from observing unboundedly high prices. In an economy with no more than 800 units of X_3, the price of any good cannot exceed 800. The highest transaction price in the observed sample occurred for P_2 = 600 in the explosive region of the CounterClockwise treatment, suggestive of an explosive tendency in prices. Regardless, though, this explosiveness is limited by the available quantity of currency in the economy.

A.2.2 Unmodeled Variation in Price Processes

While excess demand driven dynamics provide a reasonable model for expected price changes, it is an incomplete model and actual price changes are influenced by many other factors that are not included in the model. The residual price variation is apparent in the time series presented in Figure 3 and the smoothed out exponentially weighted moving average prices presented in Figure 4. Figure A.2.2 plots unsmoothed transaction prices in the price space, demonstrating substantial variation above and beyond that which is predicted by excess-demand dynamics.

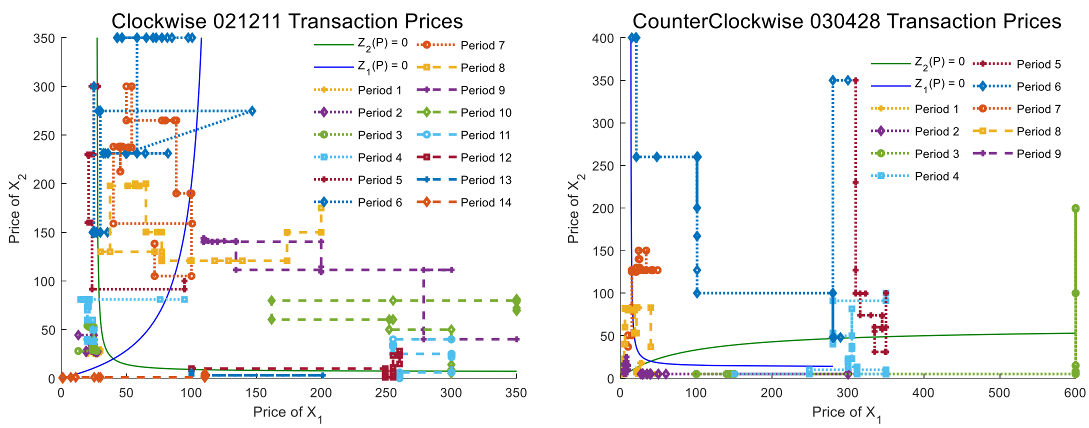

Figure A.2.2: Transaction Price Variability

Importantly, this unmodeled variability in prices is sufficient to systematically transition from explosive regions of the price space into the cyclical regions. To establish this, we consider calibrated simulations of the price process based on the estimated parameters from Table A.3.B's Aggregated Demand Model, starting from several initial positions in the explosive region while varying the amount of "noise" in the process from 25% to 100% of the estimated variance. Running 10,000 simulations for both treatments, we calculate the frequency with which the price process reaches the upper bound and the frequency with which it moves into the "Cyclical Region" as characterized by price pairs lying to the northeast of equilibrium (i.e., P_1>40 and P_2>20).

Table A.2.1 presents the results of this analysis, with essentially all simulations passing through the cyclical region and rarely reaching the price ceiling. Notably, essentially all simulations entered the Cyclical Region of price space and very rarely did the simulated price paths reach the maximum price ceilings. Across all specifications for the Clockwise treatment, only 3 out of 90,000 simulations reached the price ceiling and only 5 failed to pass through the Cyclical Region. The CounterClockwise treatment simulations had more explosive tendencies, but in the noisiest conditions less than 11% of simulations reached the price ceiling while across all specifications, over 99% of simulated price paths entered the Cyclical Region.

Table A.2.1: Properties of Simulated Price Paths

Panel A: Clockwise Treatment

	Sim Variance as % of Fitted Var	Frequency of Reaching Price Ceiling 0%	25%	100%	Frequency of Entering Cyclical Regions 0%	25%	100%
Initial Prices	(50, 10)	0%	0%	0%	0%	100%	100%
	(10, 10)	0%	0%	0%	0%	100%	100%
	(25, 25)	0%	0%	0%	0%	100%	100%

Panel B: CounterClockwise Treatment

	Sim Variance as % of Fitted Var	Frequency of Reaching Price Ceiling 0%	25%	100%	Frequency of Entering Cyclical Regions 0%	25%	100%
Initial Prices	(50, 10)	0%	0%	11%	0%	100%	100%
	(10, 10)	0%	0%	9%	0%	100%	100%
	(25, 25)	0%	0%	10%	0%	100%	100%

The price dynamics in these simulations are affected by partial equilibration forces arising from a good's own excess demand as well as general equilibration forces driven by other goods' excess demands and error correction dynamics. The influence of these additional forces on expected price dynamics is demonstrated by the simulations in which the unmodeled variance of the price process is set to zero. Notably, the price paths in this simulation never reach the price ceiling, demonstrating that general equilibration forces and error correction dynamics suffice to prevent explosive price paths. Further, these additional forces are not sufficient to drive prices into the cyclical region of price space, which requires some noise in the price process to transition phases.

A.2.3 Indivisibility, Excess Demand, and Multiple Equilibria

The unmodeled variation in price dynamics need not be entirely behavioral in its origin and could arise from approximation errors in applying a theory of equilibrium based on abstract principles to a setting that doesn't strictly satisfy all the assumptions of that theory. As an example of one such approximation error, consider the simple restriction that units of all commodities are indivisible even though the theory of price adjustment assumes individuals' consumption decisions take place on a real-valued continuum of quantity and price. Theoretically accounting for this indivisibility substantially expands the set of equilibria, as the friction associated therewith . A full analysis of these considerations presents a theoretical exercise well beyond the scope of the current paper. Our intent here is simply to demonstrate the possibility for indivisibility to generate a variety of equilibria and potential price processes.

First, we explain how to define the indivisible market excess demand as well as indivisible demand under the assumption that the commodity and price spaces are constrained to be integer-valued. Then, we demonstrate how these demand functions could influence price dynamics and market equilibria, restricting attention to the case with clockwise parameters.

A.2.3.1 Defining Integer-Valued Demand

An agent's indivisible demand function is obtained by maximizing their utility subject to a budget constraint with integer-valued variables. Let us consider the first agent-type with endowments:

$$\begin{aligned} &\max U^{(1)}(X) = \min\left[3X_2, X_3/20\right] \\ &\quad \text{subject to } P_1X_1 + P_2X_2 + X_3 = M_{10} = 800 \\ &\qquad X_i \in \mathbb{Z}_+,\ i \in \{1,2,3\} \end{aligned} \tag{A.2.1}$$

For a given integer prices (P_1, P_2), let $\hat{X}_2^{(1)}$ be the utility maximizing quantity of the second in the case of ordinary real commodity space. Ignoring the indivisibility constraint, of course this quantity will generally not be integer-valued but rather a real number. Define $\underline{X}_2 = \lfloor \hat{X}_2 \rfloor$ to denote the largest integer less than or equal to $\hat{X}_2$ and also $\overline{X}_2 = \lceil \hat{X}_2 \rceil$ denote the smallest integer larger than or equal to $\hat{X}_2$. The third good quantities demanded corresponding to $\underline{X}_2$ and $\overline{X}_2$ are respectively determined by the budget equation with $\underline{X}_3 = 800 - P_2\underline{X}_2$ and $\overline{X}_3 = 800 - P_2\overline{X}_2$, each of which will be integer-valued.

This agent is supposed to choose, as integral demand, whichever bundle of goods gives the largest utility, say, $\left(\overline{X}_2^{(1)}, \overline{X}_2^{(1)}\right)$. As demonstrated by the indifference curves in figure A.2.3, the utility from the consumption bundle $\left(0, \overline{X}_2^{(1)}, \overline{X}_3^{(1)}\right)$ is clearly larger than that derived from $\left(0, \underline{X}_2^{(1)}, \underline{X}_3^{(1)}\right)$, so the integer-valued demand vector is given by:

$$\operatorname{int} D^{(1)}(P_1, P_2) = \left[0, \overline{X}_2^{(1)}, \overline{X}_3^{(1)}\right] \tag{A.2.2}$$

A similar analysis applies to maximizing welfare and computing demand for the second agent-type that derives utility from a complementary combination of commodities X_1 and X_3.

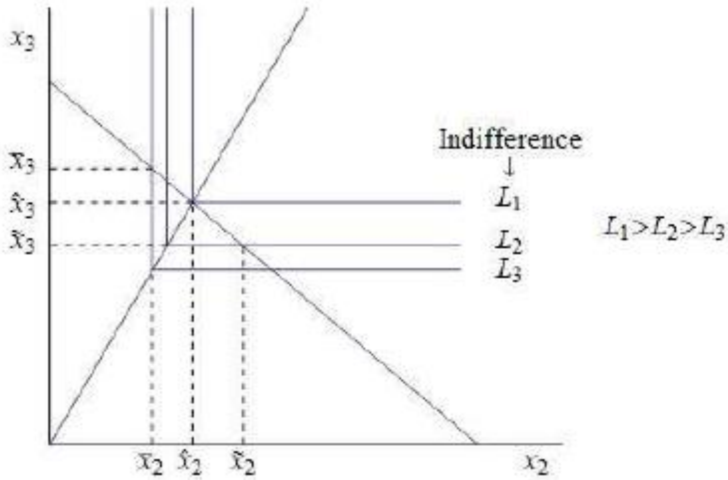

Figure A.2.3 Indifference Curves for Integer-Constrained Consumption Bundles

The third agent-type, which derives no utility from the numeraire good, presents a slightly more complicated optimization problem.

$$\begin{aligned} &\max U^3(x) = \min[3X_1, X_2/2] \\ &\quad \text{subject to } P_1X_1 + P_2X_2 + X_3 = M_3 = 40P_2 \qquad \text{(A.2.3)} \\ &\qquad\qquad X_i \in \mathbb{Z}_+,\ i \in \{1,2,3\} \end{aligned}$$

In the divisible setting of a real commodity space, the solution to this problem is uniquely determined with $X_3 = 0$. However, when restricted to integer commodity space, the solution becomes tedious and complicated. For instance, although positive holdings of X_3 is irrelevant to his utility, the integer constraint will bring about positive holding of X_3 as a result of utility maximization and could give rise to a multiplicity of solutions. How many solutions may depend on the value of relative prices for P_1 and P_2. The following picture well illustrates these phenomena.

In Figure A.2.4, let $(\hat{X}_1, \hat{X}_2)$ be the optimal consumption bundle of perfectly divisible goods and define $\underline{X}_1 = \lfloor \hat{X}_1 \rfloor$ and $\overline{X}_1 = \lceil \hat{X}_1 \rceil$ as nearest integers below and above $\hat{X}_1$, respectively as in our analysis of the first agent-type's consumption problem. Further, let $\underline{X}_2 = \lfloor 40 - P_1\underline{X}_1 / P_2 \rfloor$ and $\overline{X}_2 = \lceil 40 - P_2\overline{X}_1 / P_2 \rceil$ as the smallest and largest integers of X_2 satisfying the budget constraint. This agent's demand for good X_3 is either the residual $\underline{X}_3 = P_2(40 - \underline{X}_2) - P_1\underline{X}_1$ or $\overline{X}_3 = P_2(40 - \overline{X}_2) - P_1\overline{X}_1$. Examining the figure further illustrates the multiplicity of solutions, as the bundle $(\underline{X}_1, X_2^*)$ gives the same utility as $\left(\underline{X}_1, \underline{X}_2\right)$. Therefore, demand in this case is not actually a function, but rather the correspondence that includes, in this example:

$$\operatorname{int} D^{(3)}\left(P_1, P_2\right) \subset \left\{\left[\underline{X}_1, \underline{X}_2, \underline{X}_3\right], \left[\underline{X}_1, X_2^*, X_3^*\right]\right\} \qquad \text{(A.2.4)}$$

To get a sense of the multiplicity of such solutions, consider that for a candidate set of prices $P = (5, 400)$ there are over 70 solutions to the third agent-type's optimization problem.

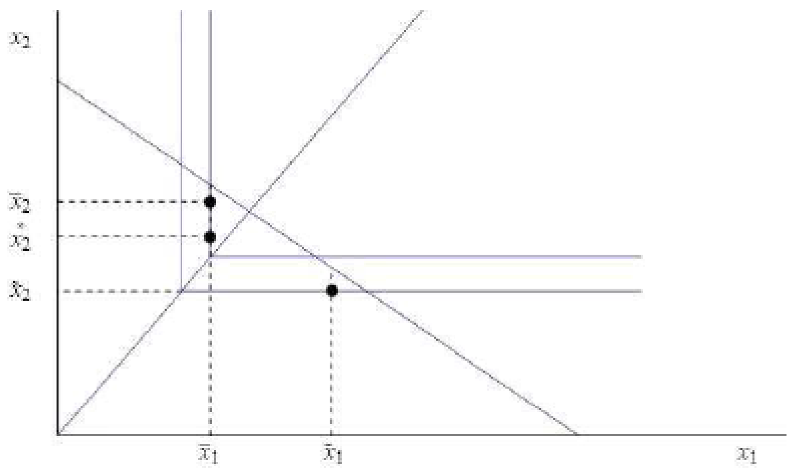

Figure A.2.4 Indifference Curves for Type-3's Integer-Constrained Consumption Bundles

Summing the demand functions for the first two agent-types with each of the candidate solutions to the third agent-type's optimization problem defines an excess demand correspondence:

$$\operatorname{int} Z\left(P_1, P_2\right) = \sum_{i=1}^{3} \operatorname{int} D^{(i)}\left(P_1, P_2\right) - \left[20,\ 40,\ 800\right] \qquad \text{(A.2.5)}$$

A.2.3.2 Equilibrium and Price Dynamics with Indivisibility

We apply brute-force numerical calculation to investigate the equilibrium and price dynamics subject to integer constraints on prices. Using Mathematica for these purposes to demonstrate our results, we consider a fixed integer price set $S = \left\{(i, j) \mid i = 1,..,200;\ j = 1,...,500\right\}$. Given the demand correspondences from the previous subsection, the equilibrium will depend on which bundle is selected from that correspondence, requiring an additional assumption to complete the model. Here, we consider one random and two deterministic approaches to resolving this indeterminacy.

First, suppose that each agent randomly draws a single consumption bundle from the correspondence that maximizes their utility for each possible price $P \in S$. Under this specification, the set of market excess demand functions becomes vast. Denoting these market excess demand functions by random-int$Z(P_1,P_2)$, not that these provide a mapping from integer space into integer space. Consequently, Walrasian dynamics can be represented by the difference equation:

$$P_t - P_{t-1} = \text{random-int}\, Z\left(p_{t-1}\right) \tag{A.2.6}$$

Given the random selection of consumption bundles, this adjustment process will be necessarily stochastic.

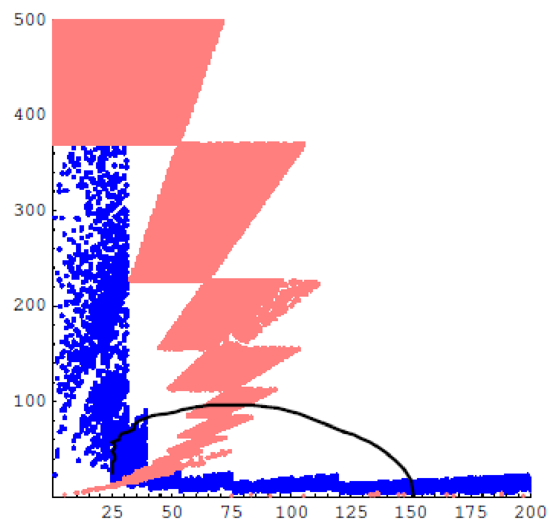

Figure A.2.5 Integer-Valued Partial Equilibrium and Excess Demand Dynamics with Randomly Selected Bundles from the Demand Correspondence

Figure A.2.5 presents the set of prices for which partial equilibrium obtains in blue pixels (for which $\text{random-int}\, Z_1\left(p\right)=0$) and red pixels (for which $\text{random-int}\, Z_2\left(p\right)=0$). Consistent with the usual definition, equilibrium prices are defined by those points at which the excess demand for both goods equals zero. Under this specification, there are 228 price combinations that are compatible with zero excess demand, demonstrating the severe multiplicity of potential equilibria. The black line traces out the price path based on the price path defined by equation (A.2.6), which notably follows the cyclical pattern despite starting in the explosive region of the clockwise treatment. Driven by the random selection consumption bundles, the integer-constrained dynamic allows for many other potential paths.

To remove the randomness in resolving the consumption choice, consider a setting wherein agents choose the bundle from their demand correspondence with either the smallest or largest quantities of X_1. These Minimal and Maximal X_1 specifications are presented in Figure A.2.6 Panels A and B, respectively. The Minimal X_1 specification in Panel A contains a large number of 964 equilibrium price combinations along with a candidate price path that travels away from equilibrium before being absorbed by the boundary. The Maximal X_1 specification in Panel B includes only 78 equilibria, and suggest a price path starting from (25, 25) that will start to orbit clockwisely before approaching the boundary price for P_2.

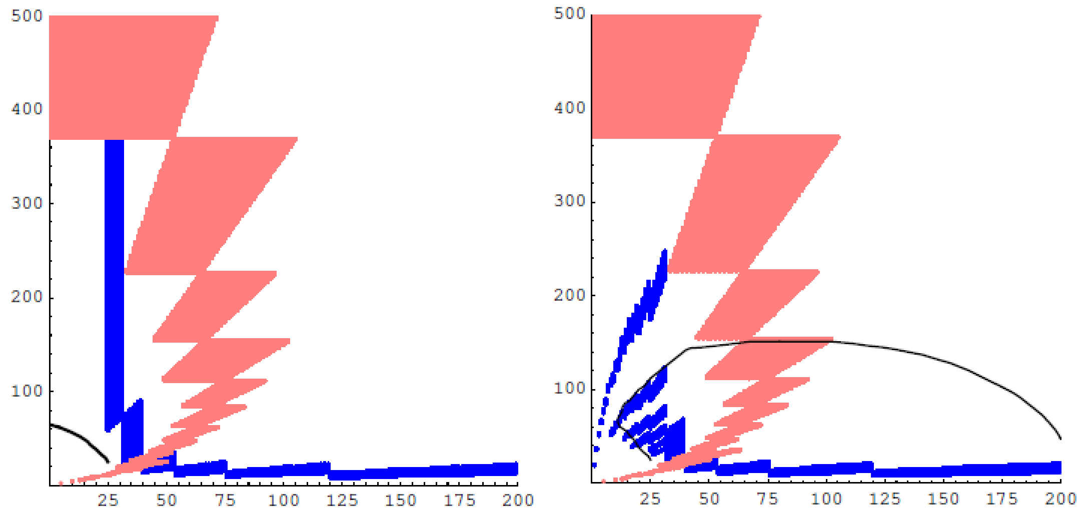

Figure A.2.6 Integer-Valued Partial Equilibrium and Excess Demand Dynamics with Selecting Bundles from the Demand Correspondence that Minimize X_1

The analysis here is in no way intended to provide an exhaustive consideration of equilibrium dynamics in the presence of indivisibility, but rather illustrates that even a simple approximating model can generate richly varied predictions after accounting for practical frictions. In many ways, this complexity underscores the surprising degree to which excess-demand driven dynamics from an abstract continuous model provide an informative device for predicting price dynamics.

A.3: Alternative Models of Disequilibrium Price Dynamics

A.3.A. Price Dynamics and Equilibrium (Non)-Attraction

In order to evaluate the degree to which equilibrium attraction might shape price dynamics, we consider whether concurrent deviations from equilibrium prices predict future price movements. To test this link, we regress changes in prices on the distance between prices and equilibrium, to which we will refer as the *Equilibrium Attraction Absolute Model*.

$$\begin{bmatrix} \dot{P}_{1,t} \\ \dot{P}_{2,t} \end{bmatrix} \equiv \begin{pmatrix} P_{1,t} - P_{1,t-1} \\ P_{2,t} - P_{2,t-1} \end{pmatrix} = \begin{bmatrix} a_{10} \\ a_{10} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} 40 - P_{1,t-1} \\ 20 - P_{2,t-1} \end{pmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \qquad \text{(A.3.1)}$$

An alternative approach to modeling price movements considers relative, rather than absolute price changes. To evaluate this specification, we regress percentage changes in prices on the distance between prices and equilibrium, by which we define the *Equilibrium Attraction Relative Model*.

$$\begin{bmatrix} \dot{P}_{1,t} / P_{1,t-1} \\ \dot{P}_{2,t} / P_{2,t-1} \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} 40 - P_{1,t-1} \\ 20 - P_{2,t-1} \end{pmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \qquad \text{(A.3.2)}$$

Table A.3.1 presents the coefficients on prices from estimating the Equilibrium Attraction model for both the absolute (Panel A) and the relative (Panel B) specifications, pooling all sessions into a common treatment.[20] Though a statistically significant relationship, the estimated impact of price deviations from equilibrium on price innovations is nearly zero. To illustrate, for every unit of X_3 that prices deviate from equilibrium prices, we would expect a correction of only 0.022 units in the next transaction.

Table A.3.1: Estimated Coefficients in Equilibrium Attraction Models

Panel A: Absolute Attraction

	Estimate	Std Error	t-Stat	p-val
a_{11}	2.22 E-02	5.38 E-03	4.21	<0.01
a_{12}	-2.28 E-03	5.41 E-03	-0.42	0.67
a_{21}	-2.50 E-03	2.41 E-03	-1.03	0.30
a_{22}	1.02 E-02	4.62 E-03	2.21	0.03

Panel B: Relative Attraction

	Estimate	Std Error	t-Stat	p-val
a_{11}	2.75 E-04	9.89 E-05	2.75	0.01
a_{12}	-4.71 E-05	1.01 E-04	-0.46	0.64
a_{21}	-1.72 E-05	7.50 E-05	-2.30	0.02
a_{22}	3.02 E-04	1.45 E-04	2.09	0.04

The Equilibrium Attraction Model, rather than serving as a theoretically grounded model of disequilibrium price dynamics, serve as an econometric specification for testing the degree to which equilibrium prices predict price changes. Indeed, the regression specifications in (A.3.1) and (A.3.2) can be interpreted as an Error Correction Model where transaction prices follow independent unit root processes converging to the equilibrium prices. Despite lacking a theoretical foundation, this specification provides a viable reduced-form device for testing whether prices' deviation from theoretical equilibrium directly predict price changes. The weakness of this predictive relationship demonstrates the degree to which prices diverge from the theoretical equilibrium.

A.3.B. Comparing Model Specifications for Price Dynamics

We can combine the regression specifications from the Equilibrium Attraction and Classical Models into an aggregated model that allows us to evaluate the relative explanatory power of Equilibrium Attraction, excess demand in the Classical Model, and

[20] We apply the same treatment to price changes as we adopt in later sections to estimate the structural relationship between price changes and excess demand. Using price changes at the transaction level, winsorized to limit outlier influence, we estimate all models equation-by-equation using FGLS accounting for Autoregressive Conditional Heteroscedasticity within each session.

excess demand in the Relative Classical Model. The regression equation of the aggregated model for commodity X_1 takes the form:

$$\begin{aligned} P_{1,t} - P_{1,t-1} = a_{10} &+ a_{11}^{EAA}\left(40 - P_{1,t-1}\right) + a_{12}^{EAA}\left(20 - P_{2,t-1}\right) \\ &+ a_{11}^{EAR}\left(40 - P_{1,t-1}\right)P_{1,t-1} + a_{12}^{EAR}\left(20 - P_{2,t-1}\right)P_{1,t-1} \\ &+ a_{11}^{GA}Z_{1,t-1}\left(P_{t-1}\right) + a_{12}^{GA}Z_{2,t-1}\left(P_{t-1}\right) \\ &+ a_{11}^{GR}Z_{1,t-1}\left(P_{t-1}\right)P_{1,t-1} + a_{12}^{GR}Z_{2,t-1}\left(P_{t-1}\right)P_{1,t-1} + \varepsilon_{1,t} \end{aligned} \qquad \text{(A.3.3)}$$

The analogous model for commodity X_2 is constructed similarly. Note that equation (A.3.3) nests all the models evaluated in the paper, with the parameters superscripted by EAA, EAR, GA, and GR corresponding to the Equilibrium Attraction Absolute, Equilibrium Attraction Relative, Generalized Absolute, and Generalized Relative Models, respectively. Our interest in this specification is purely empirical, as estimating this aggregate regression model allows us to identify which forces are most relevant to explaining price processes. Table 9 presents the regression results for the model in equation (9), separately for commodity X_1 (Panel A) and X_2 (Panel B).

Table A.3.B: Estimated Aggregated Dynamic Model Coefficients

Panel A: Commodity X_1 — *Panel B: Commodity X_2*

	Coeff	Std Error	t-Stat	p-Val		Coeff	Std Error	t-Stat	p-Val
a_{11}^{EAA}	-3.71 E-02	1.37 E-02	-0.27	0.79	a_{21}^{EAA}	1.27 E-03	5.05E-03	0.25	0.80
a_{12}^{EAA}	-1.50 E-05	7.55 E-03	-0.20	0.84	a_{22}^{EAA}	1.91 E-03	7.20E-03	0.26	0.79
a_{11}^{EAR}	9.03 E-04	4.53 E-05	1.99	0.05	a_{21}^{EAR}	-2.77E-05	1.30E-05	-2.14	0.03
a_{12}^{EAR}	-1.37 E-04	9.09 E-05	-1.50	0.13	a_{22}^{EAR}	3.30 E-05	4.31E-05	0.77	0.44
a_{11}^{GA}	3.05 E-02	2.30 E-02	1.33	0.19	a_{21}^{GA}	9.95 E-03	1.35E-02	0.74	0.46
a_{12}^{GA}	-4.60 E-03	1.49 E-02	-0.31	0.76	a_{22}^{GA}	-2.17 E-03	5.94E-03	-0.37	0.71
a_{11}^{GR}	2.68 E-03	4.79 E-04	5.59	<0.01	a_{21}^{GR}	3.93 E-04	1.77E-04	2.22	0.03
a_{12}^{GR}	4.11 E-04	1.37 E-04	2.99	<0.01	a_{22}^{GR}	2.19 E-03	2.21E-04	9.91	<0.01

We first consider the empirical relevance of the absolute models for characterizing price dynamics, which seem quite limited compared to the relative models. Only one of the eight coefficients associated with an absolute model, a_{21}^{EAA}, achieves marginal significance. However, this significance should be greeted with skepticism given the same coefficient was not statistically significant in the Equilibrium Attraction model specifications presented in Table A.3.1 that did not include excess demand measures in the set of regressors. A Wald test of the joint zero restriction on all eight absolute model coefficients is rejected at the 0.02 significance level, suggesting the absolute measures of excess demand and disequilibrium do have some explanatory power.

We next consider the degree to which equilibrium attraction forces characterize expected price dynamics. The marginal significance results indicate that a_{11}^{EAR} and a_{21}^{EAR} provide statistically significant predictors for expected price dynamics, but their influences are quite small with a price divergence of 100 leading to an expected correction of less than 1%. None of the Absolute Attraction predictors are statistically significant and a joint test that $a_{11}^{EAA} = a_{12}^{EAA} = a_{21}^{EAA} = a_{22}^{EAA} = 0$ is not rejected with a p-Value of 0.45.

Our last observation seeks to evaluate the degree to which partial and general equilibrium adjustments influence expected price dynamics. We begin by noting that the joint restriction $a_{12}^{GA} = a_{21}^{GA} = a_{12}^{GR} = a_{21}^{GR} = 0$ is rejected by the data since a_{12}^{GR} and a_{21}^{GR} both reach the threshold for statistical significance. However, the magnitude of the diagonal coefficients $\left(a_{ii}^{\square}\right)$ is clearly much greater than the magnitude of the off diagonal coefficients $\left(a_{ij}^{\square}\right)$.

APPENDIX B: Figures Presenting Time Series Results for All Sessions

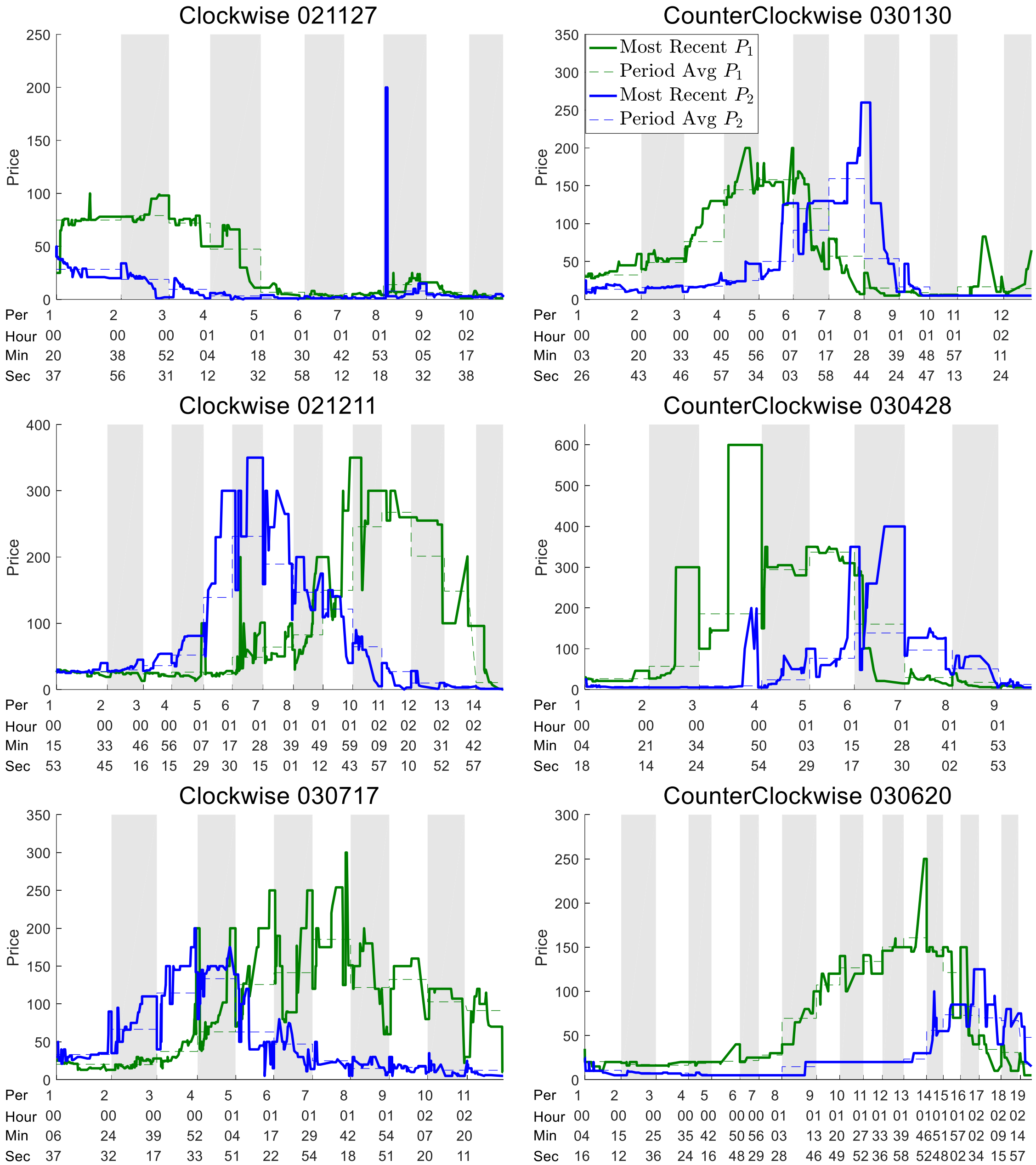

Figure B.1: Transaction Price Time Series (Figure 3)

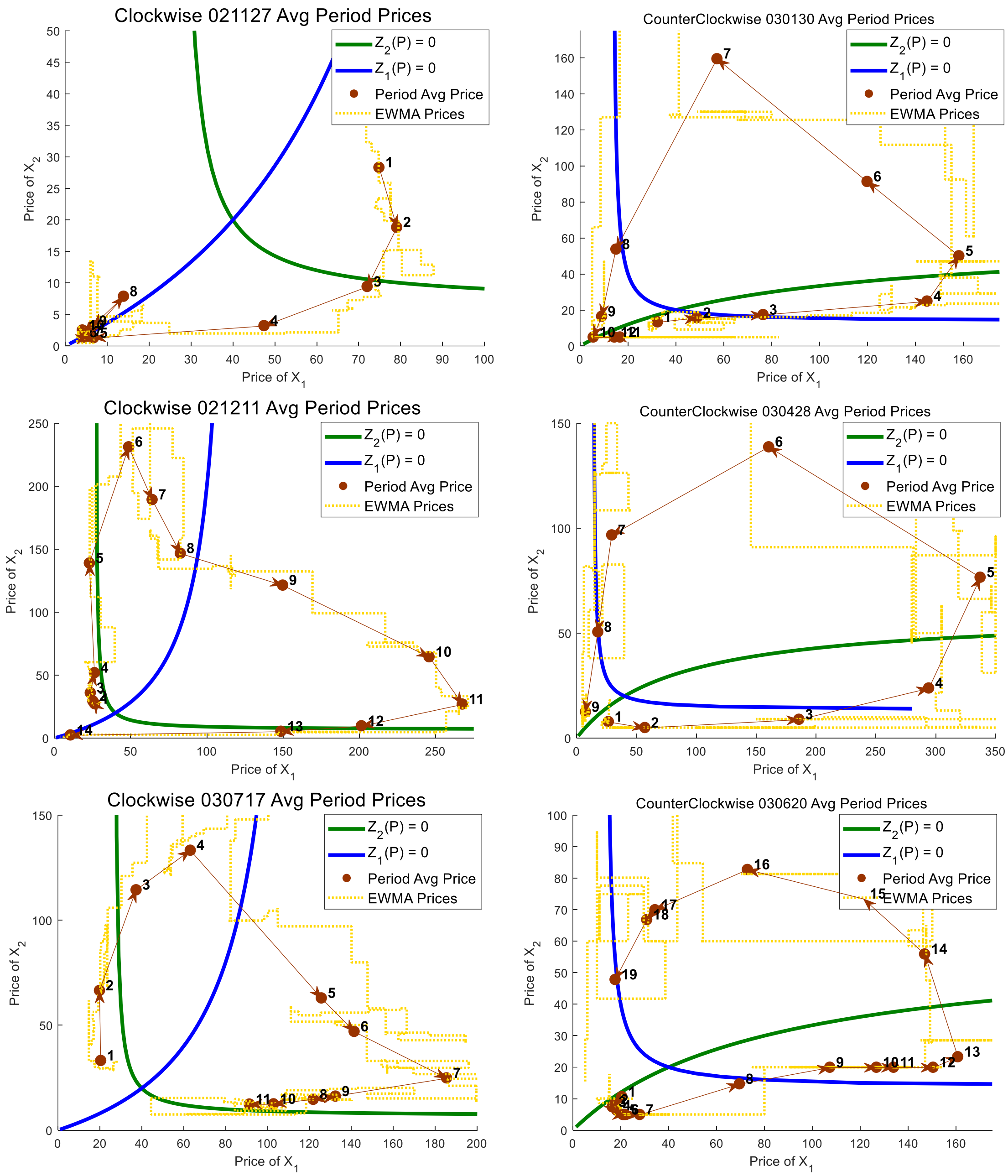

Figure B.2: Period Average Prices and Phase Diagram (Figure 4)

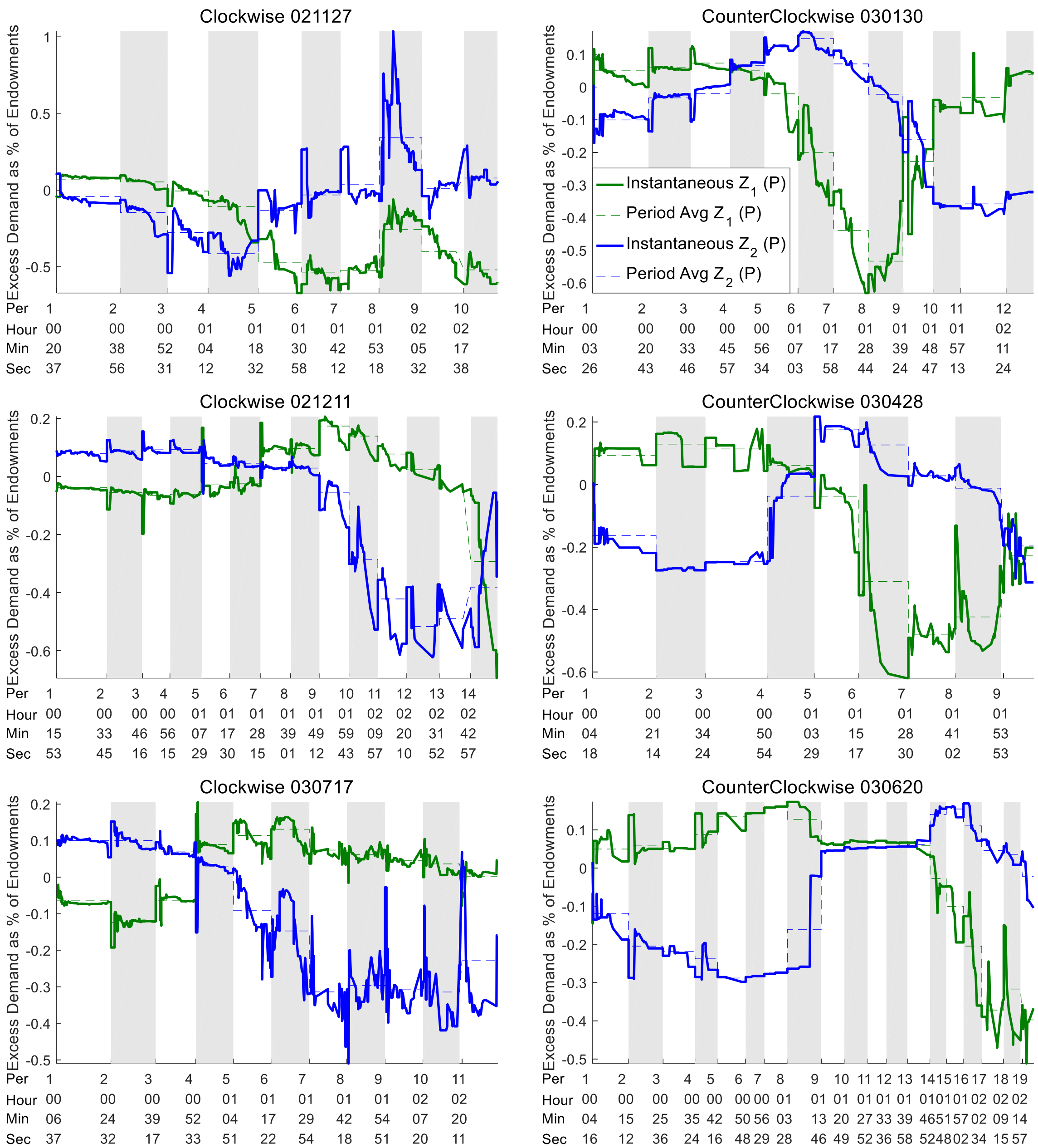

Figure B.3: Excess Demand Dynamics (Figure 6)

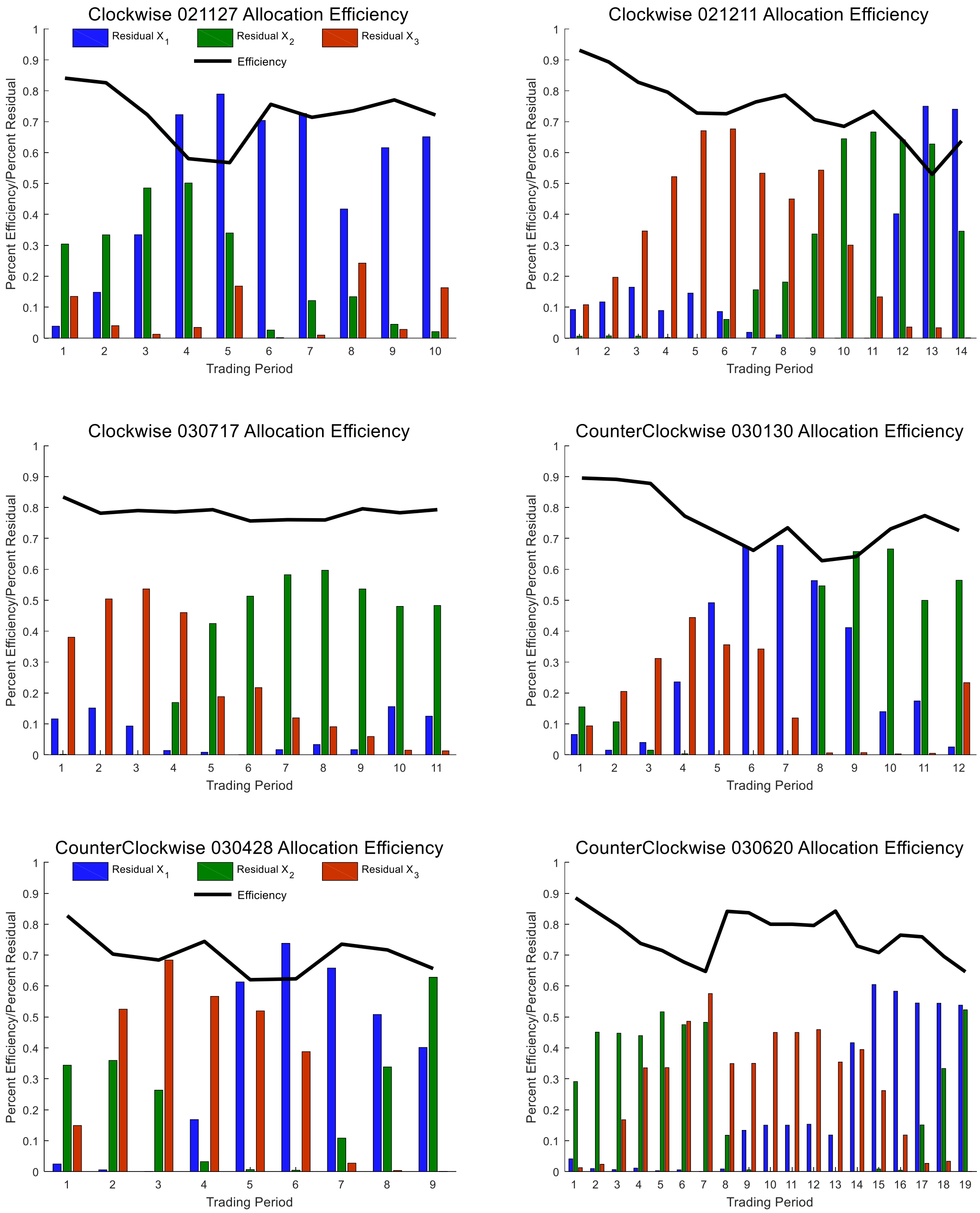

Figure B.4: Period-End Allocation Efficiency (Figure 8)

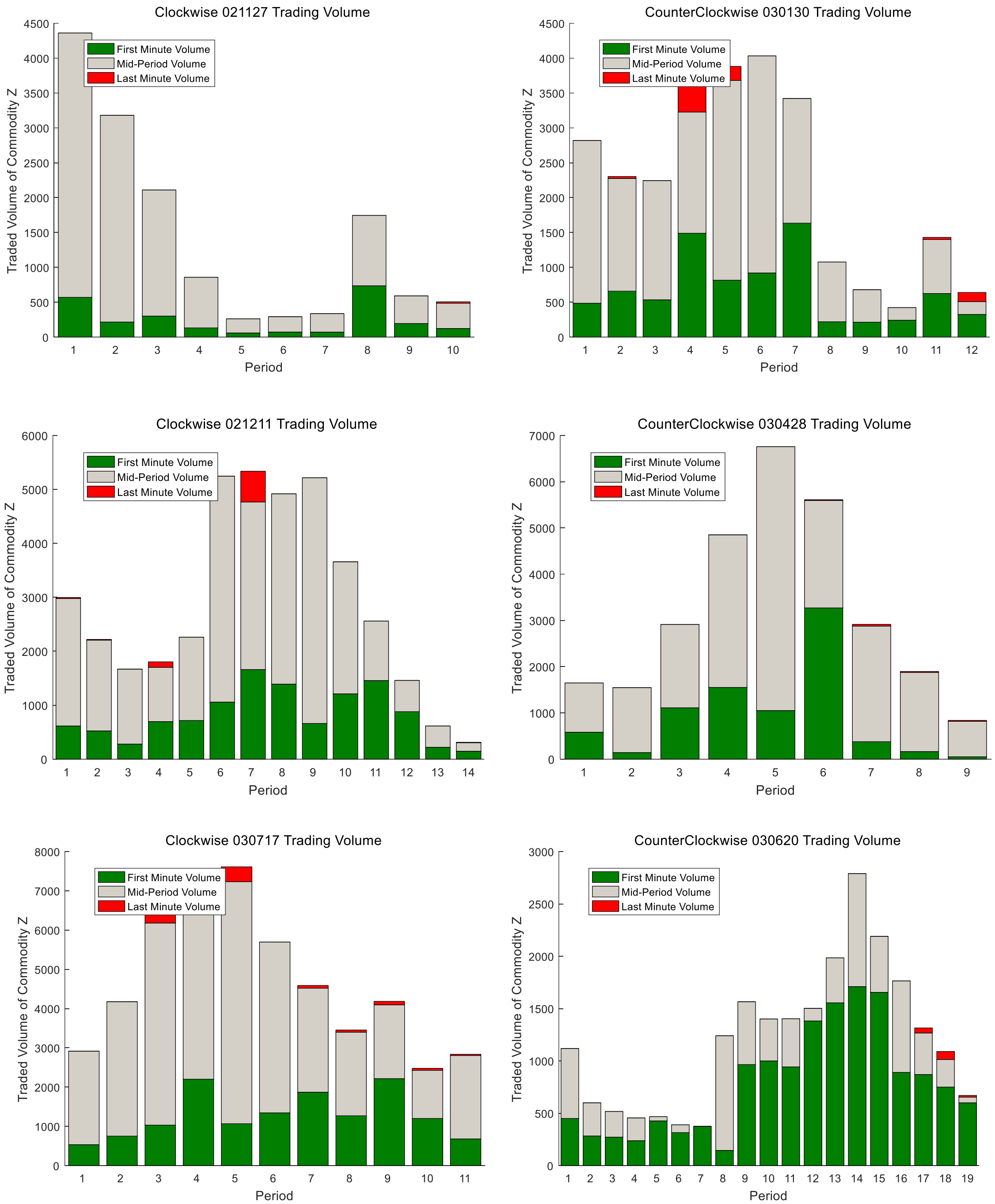

Figure B.5: Early-vs-Late Transaction Volume by Period (Figure 9)

APPENDIX C: Session-Level Regression Results

Table C.5.A: Session-Level Results for Absolute Equilibrium Attraction Model

	Clockwise - 021127				*CounterClockwise - 030130*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	5.74E-02	-1.92E-01	-4.06E-03	6.37E-02	9.68E-03	2.09E-02	-2.11E-02	7.19E-03
Std Err	1.34E-02	4.33E-02	5.15E-03	8.47E-03	1.12E-02	1.11E-02	8.13E-03	1.12E-02
t-Stat	4.28	-4.44	-0.79	7.52	0.86	1.88	-2.60	0.64

	Clockwise - 021211				*CounterClockwise - 030428*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	3.91E-02	-1.40E-02	9.42E-03	1.32E-02	1.75E-02	1.83E-02	-9.10E-03	2.36E-02
Std Err	1.44E-02	8.92E-03	6.84E-03	9.69E-03	9.79E-03	1.53E-02	4.77E-03	1.26E-02
t-Stat	2.72	-1.57	1.38	1.36	1.79	1.20	-1.91	1.88

	Clockwise - 030717				*CounterClockwise - 030620*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	3.11E-02	-1.46E-03	1.92E-02	1.14E-02	-1.77E-03	8.49E-02	-1.92E-02	2.77E-02
Std Err	1.73E-02	2.01E-02	7.14E-03	1.05E-02	1.26E-02	2.69E-02	1.17E-02	2.38E-02
t-Stat	1.80	-0.07	2.68	1.08	-0.14	3.15	-1.64	1.16

Table C.5.B: Session-Level Results for Relative Equilibrium Attraction Model

	Clockwise - 021127				*CounterClockwise - 030130*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	1.07E-03	-2.71E-03	6.78E-04	2.36E-03	5.18E-04	5.67E-04	-6.89E-04	4.04E-04
Std Err	7.18E-04	2.32E-03	4.29E-04	7.91E-04	3.54E-04	3.49E-04	2.00E-04	2.79E-04
t-Stat	1.49	-1.17	1.58	2.98	1.47	1.62	-3.45	1.45

	Clockwise - 021211				*CounterClockwise - 030428*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	2.58E-04	-3.24E-04	1.55E-04	1.97E-04	1.16E-04	2.94E-04	-3.79E-04	4.40E-04
Std Err	2.24E-04	1.38E-04	2.13E-04	3.02E-04	1.66E-04	2.58E-04	1.17E-04	3.35E-04
t-Stat	1.16	-2.36	0.73	0.65	0.70	1.14	-3.25	1.31

	Clockwise - 030717				*CounterClockwise - 030620*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	2.95E-04	-1.11E-04	2.66E-04	5.52E-04	7.69E-05	1.41E-03	-5.91E-04	5.88E-04
Std Err	1.96E-04	2.26E-04	1.98E-04	2.92E-04	2.92E-04	6.25E-04	3.32E-04	6.81E-04
t-Stat	1.51	-0.49	1.34	1.89	0.26	2.25	-1.78	0.86

Table C.7: Session-Level Results for Absolute Excess Demand Model

	Clockwise - 021127				*CounterClockwise - 030130*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	2.55E-02	1.25E-03	-1.15E-02	-1.75E-03	-5.08E-02	-6.07E-03	2.11E-02	-3.36E-04
Std Err	9.81E-03	1.42E-03	4.24E-03	5.01E-04	2.33E-02	2.30E-03	1.70E-02	1.76E-03
t-Stat	2.59	0.88	-2.71	-3.50	-2.18	-2.64	1.24	-0.19

	Clockwise - 021211				*CounterClockwise - 030428*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	2.58E-04	-3.24E-04	1.55E-04	1.97E-04	1.16E-04	2.94E-04	-3.79E-04	4.40E-04
Std Err	2.24E-04	1.38E-04	2.13E-04	3.02E-04	1.66E-04	2.58E-04	1.17E-04	3.35E-04
t-Stat	1.16	-2.36	0.73	0.65	0.70	1.14	-3.25	1.31

	Clockwise - 030717				*CounterClockwise - 030620*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	2.95E-04	-1.11E-04	2.66E-04	5.52E-04	7.69E-05	1.41E-03	-5.91E-04	5.88E-04
Std Err	1.96E-04	2.26E-04	1.98E-04	2.92E-04	2.92E-04	6.25E-04	3.32E-04	6.81E-04
t-Stat	1.51	-0.49	1.34	1.89	0.26	2.25	-1.78	0.86

Table C.8: Session-Level Results for Relative Excess Demand Model

	Clockwise - 021127				*CounterClockwise - 030130*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	8.89E-04	3.88E-05	2.86E-04	-2.98E-05	-3.34E-04	-5.46E-05	2.09E-03	-2.58E-05
Std Err	2.52E-04	1.63E-05	9.03E-04	3.53E-05	5.26E-04	2.84E-05	4.93E-04	2.99E-05
t-Stat	3.53	2.37	0.32	-0.84	-0.63	-1.92	4.24	-0.86

	Clockwise - 021211				*CounterClockwise - 030428*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	9.67E-04	2.06E-08	3.24E-04	-1.17E-05	5.29E-05	1.04E-05	1.69E-03	-1.37E-05
Std Err	2.06E-04	9.49E-06	7.89E-04	6.04E-06	2.58E-04	1.18E-05	3.61E-04	8.56E-06
t-Stat	4.70	0.00	0.41	-1.94	0.21	0.88	4.69	-1.61

	Clockwise - 030717				*CounterClockwise - 030620*			
	a_{11}	a_{12}	a_{21}	a_{22}	a_{11}	a_{12}	a_{21}	a_{22}
Coeff	3.20E-04	-1.03E-05	1.60E-03	-1.39E-05	-1.44E-03	-5.20E-05	2.65E-03	-2.84E-04
Std Err	2.60E-04	1.40E-05	7.63E-04	9.87E-06	1.38E-03	6.07E-05	1.86E-03	1.35E-04
t-Stat	1.23	-0.74	2.10	-1.41	-1.04	-0.86	1.42	-2.10

Table C.9: Session-Level Results for Aggregated Model

Panel A: Clockwise Treatment Session Level Results

Clockwise - 021127

Good X_1	a_{11}^{EAA}	a_{12}^{EAA}	a_{11}^{EAR}	a_{12}^{EAR}	a_{11}^{GA}	a_{12}^{GA}	a_{11}^{GR}	a_{12}^{GR}
Coeff	-6.12E-02	-9.53E-02	5.42E-03	2.64E-03	-2.32E-03	1.52E-02	1.55E-02	-1.61E-03
Std Err	1.42E-01	1.95E-01	1.44E-03	2.97E-03	7.06E-02	1.52E-02	3.46E-03	6.39E-04
t-Stat	-0.43	-0.49	3.77	0.89	-0.03	1.00	4.50	-2.51
Good X_2	a_{21}^{EAA}	a_{22}^{EAA}	a_{21}^{EAR}	a_{22}^{EAR}	a_{21}^{GA}	a_{22}^{GA}	a_{22}^{GA}	a_{22}^{GR}
Coeff	-1.17E-01	2.77E-01	6.86E-06	-5.16E-03	-2.85E-03	-4.25E-03	-3.31E-03	4.36E-03
Std Err	6.98E-02	1.11E-01	6.56E-04	2.25E-03	3.25E-02	7.40E-03	1.73E-03	1.47E-03
t-Stat	-1.67	2.50	0.01	-2.30	-0.09	-0.58	-1.92	2.96

Clockwise - 021211

Good X_1	a_{11}^{EAA}	a_{12}^{EAA}	a_{11}^{EAR}	a_{12}^{EAR}	a_{11}^{GA}	a_{12}^{GA}	a_{11}^{GR}	a_{12}^{GR}
Coeff	-1.33E-01	-4.18E-02	7.52E-04	7.56E-04	-2.36E-01	8.33E-02	1.22E-02	9.22E-05
Std Err	9.99E-02	1.74E-02	2.80E-04	3.69E-04	2.10E-01	1.02E-01	2.59E-03	8.62E-04
t-Stat	-1.33	-2.40	2.68	2.05	-1.12	0.82	4.70	0.11
Good X_2	a_{21}^{EAA}	a_{22}^{EAA}	a_{21}^{EAR}	a_{22}^{EAR}	a_{21}^{GA}	a_{22}^{GA}	a_{22}^{GA}	a_{22}^{GR}
Coeff	-8.89E-03	1.20E-02	-3.21E-05	-3.31E-04	1.19E-01	1.35E-02	-4.02E-03	3.30E-03
Std Err	5.38E-02	1.69E-02	1.38E-04	2.87E-04	1.85E-01	5.35E-02	1.75E-03	1.23E-03
t-Stat	-0.17	0.71	-0.23	-1.15	0.65	0.25	-2.30	2.68

Clockwise - 030717

Good X_1	a_{11}^{EAA}	a_{12}^{EAA}	a_{11}^{EAR}	a_{12}^{EAR}	a_{11}^{GA}	a_{12}^{GA}	a_{11}^{GR}	a_{12}^{GR}
Coeff	-2.20E-01	-5.80E-02	7.58E-04	1.49E-03	-6.41E-02	-3.34E-02	4.29E-03	1.44E-03
Std Err	1.16E-01	4.01E-02	4.31E-04	7.43E-04	2.53E-01	1.01E-01	3.47E-03	1.04E-03
t-Stat	-1.89	-1.45	1.76	2.00	-0.25	-0.33	1.24	1.39
Good X_2	a_{21}^{EAA}	a_{22}^{EAA}	a_{21}^{EAR}	a_{22}^{EAR}	a_{21}^{GA}	a_{22}^{GA}	a_{22}^{GA}	a_{22}^{GR}
Coeff	-8.41E-02	-3.12E-02	1.71E-04	6.19E-04	-1.36E-01	2.93E-02	1.68E-03	3.17E-03
Std Err	6.50E-02	3.57E-02	2.21E-04	3.92E-04	1.91E-01	3.95E-02	1.88E-03	1.09E-03
t-Stat	-1.29	-0.88	0.77	1.58	-0.71	0.74	0.89	2.92

Table C.9: Session-Level Results for Aggregated Model (cont.)

Panel B: CounterClockwise Treatment Session Level Results

CounterClockwise - 030130

Good X_1	a_{11}^{EAA}	a_{12}^{EAA}	a_{11}^{EAR}	a_{12}^{EAR}	a_{11}^{GA}	a_{12}^{GA}	a_{11}^{GR}	a_{12}^{GR}
Coeff	-3.31E-01	3.67E-02	2.56E-03	-1.21E-03	-2.15E-01	-1.46E-01	7.65E-03	3.29E-03
Std Err	9.61E-02	3.85E-02	6.23E-04	7.23E-04	1.12E-01	6.37E-02	2.56E-03	1.02E-03
t-Stat	-3.44	0.95	4.11	-1.68	-1.92	-2.29	2.99	3.23
Good X_2	a_{21}^{EAA}	a_{22}^{EAA}	a_{21}^{EAR}	a_{22}^{EAR}	a_{21}^{GA}	a_{22}^{GA}	a_{22}^{GA}	a_{22}^{GR}
Coeff	1.85E-02	2.82E-01	-8.02E-05	4.57E-04	2.65E-02	3.47E-02	-3.53E-03	9.58E-03
Std Err	4.85E-02	1.41E-01	2.69E-04	7.43E-04	4.81E-02	3.82E-02	2.02E-03	1.96E-03
t-Stat	0.38	2.00	-0.30	0.62	0.55	0.91	-1.75	4.90

CounterClockwise - 030421

Good X_1	a_{11}^{EAA}	a_{12}^{EAA}	a_{11}^{EAR}	a_{12}^{EAR}	a_{11}^{GA}	a_{12}^{GA}	a_{11}^{GR}	a_{12}^{GR}
Coeff	-6.92E-02	-7.08E-03	2.03E-04	-8.04E-04	-1.05E-01	-9.32E-02	7.10E-03	5.82E-04
Std Err	4.41E-02	3.04E-02	1.04E-04	2.75E-04	9.39E-02	1.10E-01	2.01E-03	3.89E-04
t-Stat	-1.57	-0.23	1.95	-2.92	-1.12	-0.84	3.53	1.50
Good X_2	a_{21}^{EAA}	a_{22}^{EAA}	a_{21}^{EAR}	a_{22}^{EAR}	a_{21}^{GA}	a_{22}^{GA}	a_{22}^{GA}	a_{22}^{GR}
Coeff	1.63E-02	-1.98E-01	-4.00E-05	4.24E-04	1.10E-02	-4.47E-03	3.04E-03	1.79E-03
Std Err	1.23E-02	9.16E-02	2.45E-05	1.86E-04	3.98E-02	3.39E-02	1.21E-03	4.62E-04
t-Stat	1.32	-2.16	-1.64	2.27	0.28	-0.13	2.51	3.87

CounterClockwise - 030620

Good X_1	a_{11}^{EAA}	a_{12}^{EAA}	a_{11}^{EAR}	a_{12}^{EAR}	a_{11}^{GA}	a_{12}^{GA}	a_{11}^{GR}	a_{12}^{GR}
Coeff	-1.68E-01	-1.38E-01	1.69E-03	1.54E-03	1.48E-01	-1.02E-01	1.14E-02	5.49E-03
Std Err	9.60E-02	1.69E-01	5.31E-04	2.61E-03	4.09E-01	2.12E-01	9.89E-03	3.51E-03
t-Stat	-1.75	-0.82	3.18	0.59	0.36	-0.48	1.15	1.57
Good X_2	a_{21}^{EAA}	a_{22}^{EAA}	a_{21}^{EAR}	a_{22}^{EAR}	a_{21}^{GA}	a_{22}^{GA}	a_{22}^{GA}	a_{22}^{GR}
Coeff	1.40E-01	-6.42E-02	-5.07E-04	7.92E-03	-6.65E-01	-1.27E-01	1.57E-02	5.72E-02
Std Err	8.23E-02	3.19E-01	5.71E-04	1.58E-03	2.49E-01	1.87E-01	9.31E-03	7.17E-03
t-Stat	1.70	-0.20	-0.89	5.02	-2.67	-0.68	1.69	7.97